

9. $\sum_{k=4}^{51} \prod_{n=1}^{15} \left(\frac{n}{n+1} \right) = ?$

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Yanıt / Answer : 3

10. $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \dots + \frac{17}{18!} = ?$

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Yanıt / Answer : $1 - \frac{1}{18!}$

11. $\prod_{k=1}^{10} \left(\frac{3}{7} \right)^k = \left(\frac{7}{3} \right)^x \Rightarrow x = ?$

Yanıt / Answer : -55

12. $\sum_{k=12}^{33} (-1)^k \cdot (2k+1) = ?$

Yanıt / Answer : -22

13. $\sum_{k=1}^{20} (3k-2) = ?$

Yanıt / Answer : 590

14. $\sum_{k=12}^{33} (-1)^k \cdot (2k+1) = ?$

Yanıt / Answer : -22

15. $\prod_{k=-2}^{11} (15-3k) = ?$

Yanıt / Answer : 0

16. $\sum_{k=6}^n \log_5 \left(1 - \frac{1}{k} \right) = -1 \Rightarrow n = ?$

Yanıt / Answer : 25

17. $\prod_{n=91}^{179} \tan x = ?$

Yanıt / Answer : -1

18. $\frac{6}{12 \cdot 13} + \frac{6}{13 \cdot 14} + \frac{6}{14 \cdot 15} + \dots + \frac{6}{24 \cdot 25} = ?$

Yanıt / Answer : $\frac{7}{26}$

19. $\sum_{k=1}^{16} \frac{1}{(2k-1)(2k+1)} = ?$

Yanıt / Answer : $\frac{16}{33}$

20. $\sum_{k=1}^{n+2} \log_3 \left(1 + \frac{1}{k}\right) = 4 \Rightarrow n = ?$

Yanıt / Answer : 78

21. $a_n = \left(1 + \frac{2n-3}{3n^2+4}\right)^{3n-1} \Rightarrow \lim(a_n) = ?$

Yanıt / Answer : e^2

22. $a_{n+1} = a_n + r, a_1 = 4,$
 $r = 4 \Rightarrow a_{18} = ?$

Yanıt / Answer : 72

23. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n = ?$

Yanıt / Answer : $-\frac{1}{3}$

24. $a_n = (a_{n-1})^r,$
 $a_1 = 4, r = \frac{1}{2} \Rightarrow a_{11} = ?$

Yanıt / Answer : $\frac{1}{2^8}$

5 5 5 5

1. $\sum_{k=1}^{20} k = ?$
 A) 120 B) 180 C) 210 D) 340 E) 420

2. $\sum_{n=3}^{18} n = ?$
 A) 165 B) 170 C) 185 D) 196 E) 204

3. $\sum_{k=3}^{27} k = ?$
 A) 300 B) 325 C) 340 D) 350 E) 375

4. $\sum_{k=1}^{16} (3k-4) = ?$
 A) 472 B) 466 C) 408 D) 344 E) 342

5. $\sum_{k=-1}^{18} 5k = ?$
 A) 750 B) 850 C) 940 D) 1050 E) 1250

6. $\sum_{k=5}^{19} (2k+1) = ?$
 A) 375 B) 350 C) 325 D) 275 E) 250

7. $\sum_{k=-2}^9 (3k+4) = ?$
 A) 124 B) 128 C) 148 D) 164 E) 174

8. $f(x) = x+3 \Rightarrow \sum_{n=1}^{10} [f(n)]^2 = ?$
 A) 705 B) 745 C) 795 D) 800 E) 805

9. $\sum_{n=1}^{70} (3+n \cdot a) = 1275 \Rightarrow a = ?$
 A) $\frac{2}{3}$ B) $\frac{3}{5}$ C) $\frac{5}{6}$ D) $\frac{5}{7}$ E) $\frac{3}{7}$

10. $\sum_{n=1}^{30} 3^n = ?$
 A) $3^{30} - 3$ B) $3^{30} + 3$ C) $\frac{3}{2}(3^{30} + 3)$
 D) $\frac{3}{2}(3^{30} - 1)$ E) $\frac{9}{2}(3^{29} - 1)$

$$11. \sum_{k=1}^{20} \frac{1}{k} - \sum_{k=1}^{20} \frac{1}{k+1} = ?$$

- A) $\frac{1}{18}$ B) $\frac{1}{20}$ C) $\frac{19}{20}$ D) $\frac{20}{21}$ E) $\frac{9}{10}$

$$12. \sum_{k=n+1}^{3n+2} \frac{1}{k^2 + k} = ?$$

- A) $\frac{2}{3(n+1)}$ B) $\frac{1}{n+3}$ C) $\frac{n}{n+1}$
D) $\frac{n}{3(n+1)}$ E) $\frac{n+1}{3n+1}$

$$13. \forall x \in \mathbb{N}^+$$

$$f(x-3) = x \Rightarrow$$

$$\sum_{n=1}^9 [f(n)]^2 = ?$$

- A) 576 B) 596 C) 600 D) 624 E) 636

$$14. \prod_{k=1}^n 3^k = 3^{300} \Rightarrow n = ?$$

- A) 36 B) 32 C) 25 D) 24 E) 20

$$15. \sum_{k=1}^4 \left[\prod_{n=1}^3 (k \cdot n!) \right] = ?$$

- A) 900 B) 1024 C) 1160 D) 1200 E) 1296

$$16. \prod_{k=2}^{63} \log_k (k+1) = ?$$

- A) 2 B) 3 C) 5 D) 6 E) 8

$$17. \sum_{k=2}^{243} \log_3 \left(1 - \frac{1}{k} \right) = ?$$

- A) 5 B) 2 C) -3 D) -5 E) -6

$$18. \sum_{k=1}^{624} \log_5 \left(1 + \frac{1}{k} \right) = ?$$

- A) 3 B) $\frac{7}{2}$ C) 4 D) 5 E) $\frac{11}{2}$

$$19. \prod_{k=1}^9 9^{\log_3 \left(1 + \frac{1}{k} \right)} = ?$$

- A) 376 B) 304 C) 196 D) 156 E) 100

$$20. \sum_{n=1}^3 \left(\prod_{m=1}^5 \frac{m \cdot n}{2} \right) = ?$$

- A) 995 B) 1015 C) 1035 D) 1105 E) 1125

Yanıtlar / Answers

1. C	2. A	3. E	4. D	5. B	6. A	7. E	8. A
9. E	10. D	11. D	12. A	13. E	14. D	15. D	16. D
17. D	18. C	19. E	20. C				

1. $\sum_{i=1}^4 \left(\sum_{j=1}^3 3j + 2i \right) = ?$

- A) 240 B) 220 C) 180 D) 170 E) 160

2. $\sum_{p=3}^6 \left(\sum_{n=0}^3 p \cdot n + 2 \right) = ?$

- A) 148 B) 144 C) 142 D) 140 E) 136

3. $\sum_{n=1}^{10} (a \cdot n - 3a) = 150 \Rightarrow a = ?$

- A) 4 B) 5 C) 6 D) 7 E) 8

4. $\sum_{n=1}^3 \left(\sum_{m=1}^3 \left(\sum_{p=1}^4 (n+2m+p) \right) \right) = ?$

- A) 280 B) 340 C) 390 D) 450 E) 490

5. $\sum_{k=15}^{20} (2k-5) = ?$

- A) 120 B) 150 C) 160 D) 180 E) 220

6. $\sum_{k=1}^{24} \frac{1}{(2k-1)(2k+1)} = ?$

- A) $\frac{1}{2}$ B) $\frac{24}{49}$ C) $\frac{25}{49}$ D) $\frac{25}{47}$ E) $\frac{3}{4}$

7. $\forall x \in \mathbb{N}^+ f(x+2) = x^2 \Rightarrow \sum_{n=5}^{12} f(n) = ?$

- A) 294 B) 298 C) 304 D) 354 E) 380

8. $\prod_{n=-9}^{12} \left(\frac{1}{3} \right)^n = ?$

- A) 3^{-33} B) 3^{-30} C) 3^{-27} D) 3^{-23} E) 3^{-18}

9. $\prod_{n=1}^{96} 2^{\log_2 \left(\frac{k+1}{k+2} \right)} = ?$

- A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{1}{4}$ D) 4 E) 8

10. $\sum_{k=1}^4 \left[\prod_{n=1}^3 (k \cdot n!) \right] = ?$

- A) 900 B) 1024 C) 1160 D) 1200 E) 1296

11. $\sum_{n=5}^9 a = 45 \Rightarrow a = ?$

- A) 5 B) 6 C) 7 D) 8 E) 9

12. $f(n) = \sum_{k=1}^n (k^2 + m - k),$

$f(1) = 4 \Rightarrow f(2) = ?$

- A) $m + 4$ B) $m + 8$ C) 14
D) 12 E) 10

13. $f: R \rightarrow R, f(x) = \sum_{k=1}^x k$
 $g: R \rightarrow R, g(x) = \sum_{k=1}^x k^2$ $\Rightarrow (g \circ f)(4) = ?$

- A) 267 B) 315 C) 340 D) 365 E) 385

14. $\prod_{k=1}^5 4^{k-2} = 32^x \Rightarrow x = ?$

- A) 5 B) 4 C) 3 D) 2 E) 1

15. $\prod_{n=1}^4 \sum_{k=1}^n \left(\frac{k}{n} \right) = ?$

- A) 16 B) 12 C) $\frac{10}{3}$ D) $\frac{15}{2}$ E) $\frac{21}{2}$

16. $\sum_{k=2}^8 (k^2 - 2k + 3) = ?$

- A) 120 B) 134 C) 154 D) 156 E) 165

17. $\sum_{n=1}^3 \left(\prod_{k=1}^4 2^k \right) = ?$

- A) 2496 B) 2966 C) 3072 D) 4642 E) 5048

18. $\prod_{n=-1}^{179} 5^{\cos(n+1)} = ?$

- A) 0 B) $\frac{1}{25}$ C) $\frac{1}{5}$ D) 1 E) 5

19. $\prod_{n=1}^{120} 5^{\log_{11} \left(1 + \frac{1}{n} \right)} = ?$

- A) $\frac{1}{25}$ B) $\frac{1}{5}$ C) 1 D) 5 E) 25

Yanıtlar / Answers

1. E	2. D	3. C	4. C	5. D	6. B	7. E	8. A
9. C	10. E	11. E	12. E	13. E	14. D	15. D	16. C
17. C	18. D	19. E					

1. $\prod_{k=5}^{124} \log_k (k+1) = ?$

- A) 2 B) 3 C) 4 D) 5 E) 6

2. $\prod_{k=1}^2 \left(\sum_{n=1}^4 (n+k) \right) = ?$

- A) 244 B) 252 C) 262 D) 276 E) 294

3. $\prod_{n=2}^5 2^{\log_2 (n+3)} = ?$

- A) 5! B) 6! C) 7! D) 8! E) 10!

4. $\prod_{n=77}^{85} 2 = ?$

- A) 64 B) 128 C) 256 D) 512 E) 1024

5. $\prod_{k=1}^n \left(1 - \frac{3}{3k+2} \right) = \frac{1}{55} \Rightarrow n = ?$

- A) 18 B) 24 C) 27 D) 32 E) 36

6. $\prod_{k=1}^{23} \sin \left[\frac{1}{24} (k+1) \pi \right] = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

7. $f: \mathbb{N}^+ \rightarrow \mathbb{N}^+, g: \mathbb{N}^+ \rightarrow \mathbb{N}^+$

$f(x) = \sum_{k=19}^x (2k-32), g(x) = \prod_{k=1}^x (3k-2)$

$\Rightarrow (f \circ g)(3) = ?$

- A) 70 B) 90 C) 120 D) 150 E) 175

8. $f: \mathbb{N}^+ \rightarrow \mathbb{N}^+,$

$f(x) = \prod_{n=1}^x 3^n \Rightarrow (f \circ f)(2) = ?$

- A) 3^{27} B) 3^{36} C) 3^{108} D) 3^{296} E) 3^{378}

9. $\sum_{n=2}^{81} \log_a \left(1 - \frac{1}{n} \right) = -2 \Rightarrow a = ?$

- A) $\frac{1}{3}$ B) $\frac{1}{9}$ C) 3 D) 9 E) 27

10. $\sum_{k=1}^{15} \frac{1}{k} - \sum_{k=1}^{15} \frac{1}{k+1} = ?$

- A) $\frac{1}{14}$ B) $\frac{1}{16}$ C) $\frac{14}{15}$ D) $\frac{5}{6}$ E) $\frac{15}{16}$

11. $\sum_{n=1}^8 2^{\log_2 n} = ?$

- A) $\log_2 315$ B) $7 + \log_2 2^{29}$ C) $5 + \log_2 63$
D) $8 + \log_3 35$ E) $12 + \log_3 7$

12. $\sum_{k=2}^{243} \log_3 \left(1 - \frac{1}{k}\right) = ?$

- A) 5 B) 2 C) -3 D) -5 E) -6

13. $\sum_{k=1}^{182} (\sqrt{4k+1} - \sqrt{4k-3}) = ?$

- A) 21 B) 24 C) 26 D) 27 E) 28

14. $\sum_{k=1}^{624} \log_5 \left(1 + \frac{1}{k}\right) = ?$

- A) 3 B) $\frac{7}{2}$ C) 4 D) 5 E) $\frac{11}{2}$

15. $\sum_{k=3}^{728} \log_3 \left(1 - \frac{1}{k+1}\right) = ?$

- A) -6 B) -5 C) -4 D) 0 E) 6

16. $\sum_{n=3}^{n-1} \log_3 \left(1 + \frac{1}{k}\right) = 6 \Rightarrow n = ?$

- A) 3^6 B) 3^7 C) 3^8 D) 3^9 E) 3^{10}

17. $\sum_{k=8}^{17} k(k!) = ?$

- A) $18! - 8!$ B) $17! - 7!$ C) $8! (10! - 1)$
D) $12! (13! - 1)$ E) $8! (9! - 1)$

18. $\frac{\prod_{k=4}^{n+3} (3k)}{(n+2)! \cdot 3^n} = ?$

- A) $n+3$ B) $3 \cdot (n+2)$ C) $\frac{n+3}{6}$
D) $\frac{n+2}{4}$ E) $\frac{(n+2) \cdot (n+3)}{3}$

19. $\sum_{n=1}^{15} (n^2 - n - 12) = ?$

- A) -1024 B) -720 C) 0 D) 940 E) 1896

Yanıtlar / Answers

1. B	2. B	3. D	4. D	5. E	6. C	7. D	8. E
9. D	10. E	11. B	12. D	13. C	14. C	15. B	16. B
17. A	18. C	19. D					

1. $(a_n) = \begin{cases} 3n-1 & n=2k \\ 5n+1 & n=2k+1 \end{cases}$

$\Rightarrow a_{11} + a_{12} = ?$

- A) 88 B) 91 C) 93 D) 95 E) 97

2. $a_1 = 4$

$a_{n+1} = a_n + n \Rightarrow a_{12} = ?$

- A) 130 B) 132 C) 136 D) 140 E) 144

3. $a_{2n-1} = \frac{4n+1}{2n-1} \Rightarrow a_n = ?$

- A) $\left(\frac{2n+2}{n}\right)$ B) $\left(\frac{2n+3}{n}\right)$ C) $\left(\frac{4n+5}{2n}\right)$
D) $\left(\frac{8n-3}{4n+3}\right)$ E) $\left(\frac{8n-7}{4n+3}\right)$

4. $b \in \mathbb{R}$

$\lim(a_n) = b$

$\lim(3(a_{n+1}) \cdot (a_{n-1}) + (a_{5n+2})) = 4 \Rightarrow b = ?$

- A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

5. $a_n = \sum_{k=1}^n \left(\frac{1}{3}\right)^k \Rightarrow \lim(a_n) = ?$

- A) $\frac{5}{2}$ B) 2 C) $\frac{3}{2}$ D) 1 E) $\frac{1}{2}$

6. $a_n = \left(\frac{3n+2}{5n-1}\right) \Rightarrow \lim(a_n) = ?$

- A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) $\frac{3}{2}$ D) $\frac{6}{5}$ E) 1

7. $(a_n) = \left(1 + \frac{3}{n+1}\right)^{2n-1} \Rightarrow \lim(a_n) = ?$

- A) e B) e^2 C) e^3 D) e^4 E) e^6

8. $a_n = a_{n-1} + r$

$a_3 = 4, a_{12} = 13 \Rightarrow a_{23} = ?$

- A) 21 B) 22 C) 23 D) 24 E) 25

9. $a_n = (a_{n-1}) \cdot r \Rightarrow a_{20} = ?$
 $a_1 = 6, r = \frac{1}{3}$

- A) $\frac{2}{3^{20}}$ B) $\frac{2}{3^{19}}$ C) $\frac{1}{3^{18}}$
D) $\frac{2}{3^{18}}$ E) $\frac{1}{3^{17}}$

10. $\sum_{n=2}^{\infty} \frac{1}{3^n} = ?$

- A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 0 E) 1

11. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = ?$

- A) $\frac{1}{6}$ B) 1 C) $\frac{3}{2}$ D) 2 E) 3

12. $S_n = \sum_{k=1}^n \frac{1}{k^2 + k} \Rightarrow \lim(S_n) = ?$

- A) 0 B) 1 C) $\frac{3}{2}$ D) 2 E) 3

13. $S_n = \sum_{k=1}^n \frac{3^k + 2^k}{5^k} \Rightarrow \lim(S_n) = ?$

- A) 2 B) $\frac{13}{6}$ C) 3 D) $\frac{17}{6}$ E) $\frac{23}{6}$

14. $(a_n) = (a_{n-1}) + r$

$a_1 = 6, r = \frac{4}{3} \Rightarrow a_{13} = ?$

- A) 22 B) 24 C) 26 D) 28 E) 30

15. $(a_n) = \frac{3^n \cdot n!}{n+1} \Rightarrow \frac{a_{n-2}}{a_{n-1}} = ?$

- A) $\frac{n!}{3(n-1)^2}$ B) $\frac{n}{3(n-1)^2}$ C) $\frac{n}{(n-1)^2}$
D) $\frac{3n}{(n-1)^2}$ E) $\frac{3}{(n-1)^2}$

16. $(a_n) = \frac{3n-1}{2n+1} \Rightarrow a_{15} = ?$

- A) $\frac{40}{31}$ B) $\frac{41}{31}$ C) $\frac{43}{31}$ D) $\frac{44}{31}$ E) $\frac{45}{30}$

17. $(a_n) = \frac{3n-2}{2n+1} \Rightarrow (a_{n+2}) = ?$

- A) $\frac{3n}{2n+3}$ B) $\frac{3n+1}{2n+1}$ C) $\frac{3n+2}{2n+2}$
D) $\frac{3n+3}{2n+3}$ E) $\frac{3n+4}{2n+5}$

18. $(a_n) = \frac{1}{(n+1)(n+2)} \Rightarrow \sum_{n=1}^{18} (a_n) = ?$

- A) $\frac{9}{20}$ B) $\frac{11}{20}$ C) $\frac{6}{10}$ D) $\frac{7}{10}$ E) $\frac{8}{20}$

Yanıtlar / Answers

1. B	2. C	3. B	4. C	5. E	6. A	7. E	8. D
9. D	10. A	11. E	12. B	13. B	14. A	15. B	16. D
17. E	18. A						

MATRİS / MATRIX

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{matrix} \text{sütun} \\ \text{column} \end{matrix}$$

→ satır / row

K keyfi bir alan, a_{ij} K içinde skalerler olmak üzere, şekildeki n tane sütun, m tane satırdan oluşan dikdörtgen tabloya **matris** denir.

*Provided that K is any area and a_{ij} are scalars in the K , the rectangle table composed of n columns and m rows in the figure is called a **matrix**.*

$$a_{ij} \in R \quad (1 \leq i \leq m, 1 \leq j \leq n)$$

m : Satır sayısı / Number of rows

n : Sütun sayısı / Number of columns

Örnek / Example:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & 1 & -3 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

A bir 3×4 matristir. A is a 3×4 matrix.

MATRİSLERİN EŞİTLİĞİ / EQUALITY of MATRICES

$$\begin{bmatrix} x & y & z \\ m & n & t \\ a & b & c \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\Rightarrow \begin{cases} x = a_{11}, & y = a_{12}, & z = a_{13} \\ m = a_{21}, & n = a_{22}, & t = a_{23} \\ a = a_{31}, & b = a_{32}, & c = a_{33} \end{cases}$$

BİR MATRİSİN bir SKALERLE ÇARPIMI
MULTIPLICATION of a MATRIX by a SCALAR

$$\alpha \in R$$

$$\alpha \cdot \begin{bmatrix} a & b & c \\ x & y & z \\ m & n & t \end{bmatrix} = \begin{bmatrix} \alpha \cdot a & \alpha \cdot b & \alpha \cdot c \\ \alpha \cdot x & \alpha \cdot y & \alpha \cdot z \\ \alpha \cdot m & \alpha \cdot n & \alpha \cdot t \end{bmatrix}$$

KARE MATRİSLER / SQUARE MATRICES

Tanım: Satır ve sütun sayısı aynı olan matrise **kare matris** denir.

Definition: A matrix having equal number of rows and columns is called a **square matrix**.

Kare Matrislere Örnekler / Examples for square matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & -4 & 1 \\ 0 & 5 & -1 & 5 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 3 & 4 & 0 \\ -1 & -7 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$



BİRİM MATRİS / UNIT MATRIX

Tanım: Köşegen (esas köşegen) üzerindeki elemanları 1 ve diğer elemanları 0 olan kare matrise **birim matris** denir. I veya I_n ile gösterilir.

Örneğin, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ matrisi üçüncü dereceden bir birim matristir.

Definition : A unit matrix is a square matrix which has every element of the main diagonal equal to 1 and all other elements equal to 0. A unit matrix is called I or I_n .

For example, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a unit matrix of order three.

Örnek / Example:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I_2 ve I_3 birim matrislerdir. / I_2 and I_3 are unit matrices.

$$IA = AI = A$$



MATRİS TOPLAMI / ADDITION of MATRICES

$$A = [a_{ij}]_{m \times n}, B = [b_{ij}]_{m \times n}$$

$$\Rightarrow A + B = [a_{ij} + b_{ij}]_{m \times n}$$

Örnek / Example:

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & 7 \\ -2 & 3 & -9 \end{bmatrix}$$

$$\Rightarrow A + B = \begin{bmatrix} 1+2 & 2-1 & -3+7 \\ 3-2 & 4+3 & 5-9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 4 \\ 1 & 7 & -4 \end{bmatrix}$$



MATRİSLERDE ÇARPMA

MULTIPLICATION in MATRICES

$$A = [a_{ij}]_{m \times n} \text{ ve (and) } B = [b_{ij}]_{n \times p}$$

A ve B matrislerinin çarpımı, $A \cdot B = AB$ biçiminde gösterilir.

Product of matrices A and B is denoted by the expressions

$$A \cdot B = AB.$$

$$A \cdot B = [a_1, a_2, \dots, a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$= [a_1 b_1 + a_2 b_2 + \dots + a_n b_n]$$

Örnek / Example:

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ -2 & 4 \\ 0 & 2 \\ -3 & 4 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & -1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ -2 & 4 \\ 0 & 2 \\ -3 & 4 \end{bmatrix}$$

$$= [6+2+0-12 \quad -2-4+6+16]$$

$$= [-4 \quad 16]$$

$$6+2+0-12 = -4, -2-4+6+16 = 16$$

Örnek / Example:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 5 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 3 \\ 4 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow A \cdot B = ?$$

Çözüm (Solution) :

$$AB = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 3 \\ 4 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

$$= \begin{matrix} & B_1 & B_2 & B_3 \\ & \uparrow & \uparrow & \uparrow \\ A_1 \leftarrow & \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 & 3 \end{bmatrix} \\ A_2 \leftarrow & \begin{bmatrix} 3 & -2 & 5 \end{bmatrix} & \begin{bmatrix} 4 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} A_1 \cdot B_1 & A_1 \cdot B_2 & A_1 \cdot B_3 \\ A_2 \cdot B_1 & A_2 \cdot B_2 & A_2 \cdot B_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-4+2 & 4-1+1 & 6+0-1 \\ 0-8+10 & 6-2+5 & 9+0-5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ 2 & 9 & 4 \end{bmatrix}$$



MATRİSLERDE ÇARPMA İŞLEMİNİN ÖZELLİKLERİ PROPERTIES of MATRICES MULTIPLICATION

1. $AB \neq BA$
2. $(AB) \cdot C = A \cdot (BC)$
3. $(B + C) \cdot A = BA + CA$
4. $k \cdot (AB) = (kA) \cdot B = A \cdot (kB) \quad (k \in \mathbb{R})$



TRANSPOZE / THE TRANSPOSE

Bir A matrisinin transpozesi A^t olarak gösterilir ve satırların yerine sütunların, sütunların yerine satırların yazılması ile elde edilir.

The transpose of a matrix A is denoted by A^t . To find the transpose of a matrix, rows of matrix are written as columns and columns of matrix are written as rows.

Örnek / Example:

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ -2 & 3 & -1 & 2 \\ 6 & -4 & -3 & 5 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 2 & -2 & 6 \\ 1 & 3 & -4 \\ 3 & -1 & -3 \\ 4 & 2 & 5 \end{bmatrix}$$



BİR MATRİSİN DETERMİNANTİ DETERMINANT of a MATRIX

Tanım: A'nın determinanı $\det A$ ya da $|A|$ ile gösterilir ve aşağıdaki şekilde tanımlanır.

Definition: Determinant of a matrix is denoted by $\det A$ or $|A|$ and is defined as the following.

$$1. A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A| = \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$2. A = \begin{bmatrix} a & b & c \\ x & y & z \\ m & n & p \end{bmatrix} \Rightarrow \det A = |A| = \begin{vmatrix} a & b & c \\ x & y & z \\ m & n & p \end{vmatrix} = (a \cdot y \cdot p + x \cdot n \cdot c + m \cdot b \cdot z) - (m \cdot y \cdot c + a \cdot n \cdot z + x \cdot b \cdot p)$$

Örnek / Example:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow |A| = ?$$

Çözüm / Solution:

$$|A| = 3(12) + 0(6 - 0) - 0(5 - 8)$$

$$|A| = 3 \cdot 12 = 36$$

$$|A| = 3 \cdot 2 \cdot 6 = 36$$

Örnek / Example:

$$A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & 0 \\ -1 & 2 & -3 \\ 2 & 3 & 4 \end{bmatrix}$$

$$|A| = -1 \cdot (4 - 0) - 2 \cdot (16 - 0) - 3 \cdot (12 - 2)$$

$$|A| = -4 - 32 - 30 = -66$$

$$|B| = 4 \cdot (8 + 9) - 1 \cdot (-4 + 6) + 0 \cdot (12 - 2)$$

$$|B| = 68 - 2 = 66$$

$$|B| = -|A| = (-66) = 66$$

☞ Bir determinantın karşılıklı iki satır (veya sütun) elemanları orantılı ise $|A| = 0$ dir.

If the elements on two rows (or columns) are proportional with each other in a matrix then $|A| = 0$.



BİR MATRİSİN TERSİ / INVERSE of a MATRIX

Bir matrisin tersi A^{-1} ile gösterilir.

Inverse of a matrix is denoted by A^{-1}

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det |A^{-1}| = (\det A)^{-1} = \frac{1}{\det A}$$

Örnek / Example:

$$A = \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} \Rightarrow A^{-1} = ?$$

Çözüm / Solution:

$$|A| = 12 - 20 = -8$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 6 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{3}{4} & \frac{5}{8} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

Örnek / Example:

$$\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = I_{2 \times 2} \Rightarrow a + b + c + d = ?$$

Çözüm / Solution:

$$|A| = 21 - 20 = 1$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$$

$$a + b + c + d = 19$$

ÇÖZÜMLÜ TEST / TEST WITH SOLUTIONS

$$1. \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 & 4 \\ 2 & -4 & 3 \end{bmatrix} \Rightarrow A+B = ?$$

$$A) \begin{bmatrix} 0 & 3 & 7 \\ 5 & 0 & 4 \end{bmatrix}$$

$$B) \begin{bmatrix} 0 & 3 & 7 \\ 3 & 4 & 1 \end{bmatrix}$$

$$C) \begin{bmatrix} 0 & 3 & 7 \\ 2 & -4 & 3 \end{bmatrix}$$

$$D) \begin{bmatrix} 0 & 3 & 3 \\ 2 & -4 & 1 \end{bmatrix}$$

$$E) \begin{bmatrix} 0 & 3 & 4 \\ 5 & 0 & 7 \end{bmatrix}$$

Çözüm / Solution:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 4 \\ 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 7 \\ 5 & 0 & 4 \end{bmatrix}$$

Yanıt / Answer A

$$2. \quad A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix} \Rightarrow A+2B = ?$$

$$A) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B) \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$$

$$C) \begin{bmatrix} 0 & 1 \\ 7 & 0 \end{bmatrix}$$

$$D) \begin{bmatrix} 3 & -4 \\ 8 & 4 \end{bmatrix}$$

$$E) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Çözüm / Solution:

$$\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} + 2 \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 7 & 0 \end{bmatrix}$$

Yanıt / Answer C

$$3. \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \Rightarrow$$

$$2a + b + c - 2d = ?$$

- A) -3 B) -2 C) -1 D) 0 E) 1

Çözüm / Solution:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} a+5 & b+6 \\ c+7 & d+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$a+5=6 \Rightarrow a=1$$

$$b+6=8 \Rightarrow b=2$$

$$c+7=10 \Rightarrow c=3$$

$$d+8=12 \Rightarrow d=4$$

$$2a+b+c-2d=2 \cdot 1+2+3-2 \cdot 4=-1$$

Yanıt / Answer C

$$4. \quad A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x + 2A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow x = ?$$

A) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$ C) $\begin{bmatrix} -1 & -2 \\ -1 & -2 \end{bmatrix}$

D) $\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$ E) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Çözüm / Solution:

$$x = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$a+1=0 \Rightarrow a=-1$$

$$b+2=0 \Rightarrow b=-2$$

$$c+2=0 \Rightarrow c=-2$$

$$d+1=0 \Rightarrow d=-1$$

$$x = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$$

Yanıt / Answer D

$$5. \quad A = \begin{bmatrix} 3 & -1 \\ 4 & 4 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\Rightarrow A \cdot B = ?$$

A) $\begin{bmatrix} 1 & 15 & 6 \\ 5 & 28 & 16 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 5 \\ 12 & 28 \\ 8 & 18 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 5 \\ 12 & 18 \\ 6 & 18 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 12 & 8 \\ 5 & 28 & 18 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 5 \\ 12 & 28 \\ 18 & 8 \end{bmatrix}$

Çözüm / Solution:

$$\begin{bmatrix} 3 & -1 \\ 4 & 4 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 1 + (-1) \cdot 2 & 3 \cdot 3 + (-1) \cdot 4 \\ 4 \cdot 1 + 4 \cdot 2 & 4 \cdot 3 + 4 \cdot 4 \\ 2 \cdot 1 + 3 \cdot 2 & 2 \cdot 3 + 3 \cdot 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 12 & 28 \\ 8 & 18 \end{bmatrix}$$

Yanıt / Answer B

$$6. \quad A = \begin{bmatrix} \sqrt{8} & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2^y & \log_4 x \\ 1 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} \sqrt{32} & \log_9 27 \\ 1 & 2 \end{bmatrix}$$

$$A+B=C \Rightarrow x \cdot y = ?$$

- A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution:

$$A+B=C$$

$$\begin{bmatrix} \sqrt{8} & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2^y & \log_4 x \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{32} & \log_9 27 \\ 1 & 2 \end{bmatrix}$$

$$\sqrt{8} + 2^y = \sqrt{32} \Rightarrow 2^y = 4\sqrt{2} - 2\sqrt{2}$$

$$2^y = 2\sqrt{2}$$

$$2^y = 2^{\frac{3}{2}}$$

$$y = \frac{3}{2}$$

$$1 + \log_4 x = \log_9 27 = \log_{3^2} 3^3$$

$$1 + \log_4 x = \frac{3}{2}$$

$$\log_4 x = \frac{3}{2} - 1 = \frac{1}{2} \Rightarrow x = 4^{\frac{1}{2}} = 2 \Rightarrow x = 2$$

$$x \cdot y = 2 \cdot \frac{3}{2} = 3$$

Yanıt / Answer C

$$7. A = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix} \Rightarrow A^{-1} = ?$$

$$A) \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix} \quad B) \begin{bmatrix} 4 & 1 \\ 11 & 3 \end{bmatrix} \quad C) \begin{bmatrix} 4 & 1 \\ -11 & 3 \end{bmatrix}$$

$$D) \begin{bmatrix} 2 & -\frac{1}{2} \\ -\frac{11}{2} & \frac{3}{2} \end{bmatrix} \quad E) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Çözüm / Solution:

$$A = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix}$$

$$\det(A) = 3 \cdot 4 - 11 \cdot 1 = 1$$

$$A^{-1} = 1 \cdot \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix}$$

Yanıt / Answer A

$$8. A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \Rightarrow A + A^T = ?$$

$$A) \begin{bmatrix} 6 & 1 \\ 1 & 3 \end{bmatrix} \quad B) \begin{bmatrix} 6 & 1 \\ 1 & 4 \end{bmatrix} \quad C) \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$$

$$D) \begin{bmatrix} 6 & 1 \\ 0 & 4 \end{bmatrix} \quad E) \begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix}$$

Çözüm / Solution:

$$A^T = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 1 & 4 \end{bmatrix}$$

Yanıt / Answer B

$$9. A = \begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^{16} = ?$$

$$A) 2^8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B) 2^{16} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad C) 2^{32} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$D) 2^{16} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad E) 2^{32} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Çözüm / Solution:

$$A^2 = A \cdot A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 \\ 0 & 2^2 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} = \begin{bmatrix} 2^4 & 0 \\ 0 & 2^4 \end{bmatrix}$$

$$A^8 = A^4 \cdot A^4 = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix} = \begin{bmatrix} 2^8 & 0 \\ 0 & 2^8 \end{bmatrix}$$

$$A^{16} = A^8 \cdot A^8 = \begin{bmatrix} 2^{16} & 0 \\ 0 & 2^{16} \end{bmatrix} = 2^{16} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Yanıt / Answer D

$$10. A = \begin{vmatrix} x+2y & x+y \\ x-2y & x-y \end{vmatrix} \Rightarrow$$

$$\det(A) = ?$$

$$A) xy \quad B) x \quad C) -2y \quad D) y \quad E) 2xy$$

Çözüm / Solution:

$$\det(A) = (x+2y) \cdot (x-y) - (x-2y) \cdot (x+y)$$

$$= x^2 - xy + 2xy - 2y^2 - x^2 - xy + 2xy + 2y^2$$

$$= 2xy$$

Yanıt / Answer E

11. $A = \begin{vmatrix} x+1 & 3 & -2 \\ 4 & 1 & 0 \\ 3 & x & 1 \end{vmatrix} = 9 \Rightarrow x = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution:

$$\begin{vmatrix} x+1 & 3 & -2 \\ 4 & 1 & 0 \\ 3 & x & 1 \end{vmatrix} = (x+1) \cdot 1 \cdot 1 + (-2) \cdot x \cdot 4 + 0 \cdot 3 \cdot 3 - [3 \cdot 1 \cdot (-2) + (x+1) \cdot x \cdot 0 + 4 \cdot 3 \cdot 1] = 9$$

$$\begin{aligned} (x+1) - 8x - (-6 + 12) &= 9 \\ (x+1) - 8x - 6 + 12 &= 9 \\ -7x + 1 - 6 + 12 &= 9 \\ -7x + 7 &= 9 \\ -7x &= 2 \\ 7x &= -2 \end{aligned}$$

Yanıt / Answer A

12. $\begin{vmatrix} 0 & 1 & x-2 \\ 0 & -1 & 3 \\ x+2 & 0 & 1 \end{vmatrix} = 0$

Denkleminin çözüm kümesi hangisidir?

What is the solution set for $x = ?$

- A) $\{-1\}$ B) $\{1\}$ C) $\{1, 2\}$
D) $\{-2\}$ E) $\{-2, -1\}$

Çözüm / Solution:

$$\begin{vmatrix} 0 & 1 & x-2 \\ 0 & -1 & 3 \\ x+2 & 0 & 1 \end{vmatrix} = 0 \Rightarrow 3(x+2) + (x+2) \cdot (x-2) = 0$$

$$(x+2)(3+x-2) = 0$$

$$(x+2)(1+x) = 0$$

$$x_1 = -2$$

$$x_2 = -1$$

$$\{-2, -1\}$$

Yanıt / Answer E

YÖS SORULARI / YÖS QUESTIONS

1. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = ?$

- A) $\begin{bmatrix} 7 & 8 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$
D) $\begin{bmatrix} 8 \\ 7 \end{bmatrix}$ E) $\begin{bmatrix} 7 \\ 8 \end{bmatrix}$

(YÖS 1990)

Çözüm / Solution:

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 3 \\ 4 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 & 3 \\ 4 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Yanıt / Answer E

2. $\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}^T = ?$

- A) $\begin{bmatrix} 3 & 3 & 2 \\ 2 & 2 & 0 \\ 2 & 2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$
D) $\begin{bmatrix} 3 & 3 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 1 \\ 3 & 1 \\ 2 & 2 \end{bmatrix}$

(YÖS 1991)

Çözüm / Solution:

$$\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 2 \\ 2 & 2 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

Yanıt / Answer A

3. $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1} = ?$

A) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ B) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

D) $\begin{bmatrix} -3 & -5 \\ -1 & -2 \end{bmatrix}$ E) $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

(YÖS 1992)

Çözüm / Solution:

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{3 \cdot 2 - 5 \cdot 1} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$= \frac{1}{1} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

Yanıt / Answer E

4. $\left[\frac{3!}{(x-1)!} \right] = \left[\frac{4!}{x \cdot 2y+2} \right] \Rightarrow x \cdot y = ?$

A) 8 B) 6 C) 5 D) 4 E) 2

(YÖS 1993)

Çözüm / Solution:

$$3! = \frac{4!}{x} \Rightarrow 3! = \frac{4 \cdot 3!}{x} \Rightarrow x = 4$$

$$(x-1)! = 2y+2 \Rightarrow 3! = 2y+2$$

$$6-2=2y$$

$$2=y$$

$$x \cdot y = 4 \cdot 2$$

$$= 8$$

Yanıt / Answer A

5. $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \Rightarrow A^2 = ?$

A) $\begin{bmatrix} -1 & \sin 2\alpha \\ \sin 2\alpha & \sin \alpha \end{bmatrix}$ B) $\begin{bmatrix} 1 & 1 \\ \sin 2\alpha & \cos 2 \end{bmatrix}$

C) $\begin{bmatrix} 2 & \cos \alpha \\ \cos \alpha & 2 \end{bmatrix}$ D) $\begin{bmatrix} 1 & \cos \alpha \\ \sin \alpha & 1 \end{bmatrix}$

E) $\begin{bmatrix} 1 & \sin 2\alpha \\ \sin 2\alpha & 1 \end{bmatrix}$

(YÖS 1993)

Çözüm / Solution:

$$A^2 = \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \cdot \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \sin 2\alpha \\ \sin 2\alpha & 1 \end{bmatrix}$$

Yanıt / Answer E

6. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A^n = \begin{bmatrix} 32 & 32 \\ 32 & 32 \end{bmatrix} \Rightarrow n = ?$

A) 1 B) 3 C) 4 D) 5 E) 6

(YÖS 1994)

Çözüm / Solution:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^n = 2^{n-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^n = 2^5 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow n-1=5$$

$$n=6$$

Yanıt / Answer E

7. $A = \begin{bmatrix} 2y & 2(y^2 + y + 1) \\ 2 & 2(y+1) \end{bmatrix} \Rightarrow \det A = ?$

- A) -8 B) -4 C) -1 D) 1 E) 2

(YÖS 1995)

Çözüm / Solution:

$$A = \begin{bmatrix} 2y & 2(y^2 + y + 1) \\ 2 & 2(y+1) \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \det A &= 2y \cdot 2(y+1) - 2 \cdot 2 \cdot (y^2 + y + 1) \\ &= 4y^2 + 4y - 4y^2 - 4y - 4 \\ &= -4 \end{aligned}$$

Yanıt / Answer B

8. $\begin{bmatrix} 10 & -5 \\ 6 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} x & y \\ z & t \end{bmatrix} \Rightarrow \begin{bmatrix} x & y \\ z & t \end{bmatrix} = ?$

A) $\begin{bmatrix} 30 & -20 \\ -24 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 30 & 20 \\ 24 & -6 \end{bmatrix}$

C) $\begin{bmatrix} -10 & 20 \\ 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} -10 & -40 \\ 0 & -24 \end{bmatrix}$

E) $\begin{bmatrix} 5 & 30 \\ 0 & 10 \end{bmatrix}$

(YÖS 1997)

Çözüm / Solution:

$$\begin{aligned} x &= 10 \cdot 2 - 5 \cdot 6 = -10 \\ y &= 10 \cdot (-4) + 0 = -40 \\ z &= 6 \cdot 2 - 2 \cdot 6 = 0 \\ t &= 6 \cdot (-4) + 0 = -24 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} -10 & -40 \\ 0 & -24 \end{bmatrix}$$

Yanıt / Answer D

9. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & a & 10 \\ -10 & 0 & 2 \end{vmatrix} = -56 \Rightarrow a^2 - 2a + 1 = ?$

- A) 11 B) 13 C) 14 D) 16 E) 17

(YÖS 1997)

Çözüm (Solution) :

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & a & 10 \\ -10 & 0 & 2 \end{vmatrix} = -56$$

$$2a + 10 - 200 - (-30a + 0 + 16) = -56$$

$$2a - 200 + 30a - 16 = -56$$

$$32a = 160$$

$$a = 5$$

$$a^2 - 2a + 1 = (a-1)^2 = (5-1)^2 = 16$$

Yanıt / Answer D

10. $\begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix} \cdot \begin{bmatrix} a & a \\ 1 & b \end{bmatrix} = \begin{bmatrix} 33 & 33 \\ 77 & 77 \end{bmatrix}$

$$\Rightarrow a + b = ?$$

- A) 7 B) 8 C) 9 D) 10 E) 11

(YÖS 1998)

Çözüm / Solution:

$$\begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix} \cdot \begin{bmatrix} a & a \\ 1 & b \end{bmatrix} = \begin{bmatrix} 3a+3 & 3a+3b \\ 7a+7 & 7a+7b \end{bmatrix} = \begin{bmatrix} 33 & 33 \\ 77 & 77 \end{bmatrix}$$

$$3a+3=33 \Rightarrow 3a=30 \Rightarrow a=10$$

$$3a+3b=33 \Rightarrow 30+3b=33 \Rightarrow b=1$$

$$a+b=10+1=11$$

Yanıt / Answer E

11. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix} \Rightarrow x+y+z = ?$

- A) 4 B) 5 C) 6 D) 7 E) 8

(YÖS 2001)

Çözüm / Solution:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \cdot x + 0 \cdot y + 0 \cdot z \\ 0 \cdot x + 2 \cdot y + 0 \cdot z \\ 0 \cdot x + 0 \cdot y + 3 \cdot z \end{bmatrix} = \begin{bmatrix} x \\ 2y \\ 3z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ 2y \\ 3z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix} \Rightarrow \begin{aligned} x &= 3 \\ y &= 1 \\ z &= 2 \end{aligned} \Rightarrow x+y+z = 6$$

Yanıt / Answer C

12. $0 < a$

$$A = \begin{bmatrix} 2 & a & 4 \\ a & 0 & -a \\ 0 & 5 & -1 \end{bmatrix} \det A = 400$$

$\Rightarrow a = ?$

- A) 10 B) 12 C) 18 D) 30 E) 40

(YÖS 2001)

Çözüm / Solution:

$$= [2 \cdot 0 \cdot (-1) + a \cdot 5 \cdot 4 + 0 \cdot a \cdot (-a)] - [4 \cdot 0 \cdot 0 + (-a) \cdot 5 \cdot 2 + (-1) \cdot a \cdot a]$$

$$= 20a + 10a + a^2 = 400$$

$$\Rightarrow a^2 + 30a - 400 = 0 \Rightarrow (a + 40)(a - 10) = 0$$

$$a = 10$$

Yanıt / Answer A

13. $\begin{vmatrix} 2005 & 2001 \\ 2001 & 2005 \end{vmatrix} = ?$

- A) 16076 B) 16024 C) 16016
D) 16000 E) 15976

(YÖS 2002)

Çözüm / Solution:

$$\begin{vmatrix} 2005 & 2001 \\ 2001 & 2005 \end{vmatrix} = 2005^2 - 2001^2$$

$$= (2005 - 2001)(2005 + 2001)$$

$$= 4 \cdot 4006$$

$$= 16024$$

Yanıt - Answer B

14. $A = \begin{bmatrix} 2 & -3 \\ 1 & a \end{bmatrix}, A^T \cdot A = \begin{bmatrix} 5 & -2 \\ -2 & 25 \end{bmatrix} \Rightarrow a = ?$

- A) 2 B) 3 C) 4
D) 5 E) 6

(YÖS 2003)

Çözüm / Solution:

$$A^T = \begin{bmatrix} 2 & 1 \\ -3 & a \end{bmatrix} \Rightarrow A^T \cdot A = \begin{bmatrix} 2 & 1 \\ -3 & a \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 1 & a \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 2 + 1 \cdot 1 & 2 \cdot (-3) + 1 \cdot a \\ (-3) \cdot 2 + a \cdot 1 & (-3) \cdot (-3) + a \cdot a \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -6 + a \\ -6 + a & 9 + a^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & -6 + a \\ -6 + a & 9 + a^2 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 25 \end{bmatrix}$$

$$\Rightarrow -6 + a = -2$$

$$a = 4$$

Yanıt / Answer C

15. $\begin{vmatrix} 20 & 21 & 3 \\ 30 & 32 & 6 \\ 40 & 43 & a \end{vmatrix} = 0 \Rightarrow a = ?$

- A) 9 B) 8 C) 7
D) 6 E) 5

(YÖS 2003)

Çözüm / Solution:

$$\begin{vmatrix} 40 & 42 & 6 \\ 30 & 32 & 6 \\ 40 & 43 & a \end{vmatrix}, \begin{vmatrix} 0 & -1 & 6-a \\ 30 & 32 & 6 \\ 40 & 43 & a \end{vmatrix} = 0$$

$$1 \cdot |30a - 240| + (6 - a) \cdot |1290 - 40 \cdot 32| = 0$$

$$30a - 240 + (6 - a) \cdot 10(129 - 128) = 0$$

$$30a - 240 + 60 - 10a = 0$$

$$20a = 180$$

$$a = 9$$

Yanıt / Answer A

16. $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} = A \cdot \begin{bmatrix} 2 & 3 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \Rightarrow 11 \cdot A = ?$

A) $\begin{bmatrix} 5 & 7 \\ 4 & 10 \end{bmatrix}$ B) $\begin{bmatrix} -5 & 7 \\ 4 & 10 \end{bmatrix}$ C) $\begin{bmatrix} -5 & 7 \\ 10 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 5 & -7 \\ 4 & -10 \end{bmatrix}$ E) $\begin{bmatrix} 5 & 7 \\ 10 & 4 \end{bmatrix}$

(YÖS 2004)

Çözüm / Solution:

$$\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = A \cdot \begin{bmatrix} 2 & 3 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ -2 & 2 \end{bmatrix} = A \cdot \begin{bmatrix} 2 & 3 \\ -3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -3 & 1 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} -1 & 2 \\ -2 & 2 \end{bmatrix} \cdot \frac{1}{11} \cdot \begin{bmatrix} 1 & -3 \\ 3 & 2 \end{bmatrix}$$

$$11A = \begin{bmatrix} -1+6 & 3+4 \\ -2+6 & 6+4 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 4 & 10 \end{bmatrix}$$

Yanıt / Answer A

$$17. \left| \begin{array}{cc} \log_3 4 & -3 \\ 5 & \log_4 9 \end{array} \right| = ?$$

- A) 10 B) 12 C) 13 D) 16 E) 17

(YÖS 2004)

Çözüm / Solution:

$$\left| \begin{array}{cc} \log_3 4 & -3 \\ 5 & \log_4 9 \end{array} \right| = \log_3 4 \cdot \log_4 9 - (-3) \cdot 5$$

$$= \log_3 9 + 15 = 2 + 15 = 17$$

Yanıt / Answer E

$$18. \begin{bmatrix} 7 & c \\ 3 & a-b \end{bmatrix} = \begin{bmatrix} a+b & 2 \\ d-c & 1 \end{bmatrix}$$

$$a \cdot b \cdot c \cdot d = ?$$

- A) 20 B) 40 C) 80 D) 100 E) 120

(YÖS 2005)

Çözüm / Solution:

$$7 = a + b \quad 3 = d - c$$

$$c = 2 \quad a - b = 1$$

$$\begin{cases} a+b=7 \\ a-b=1 \end{cases} \Rightarrow a=4, b=3$$

$$c=2 \Rightarrow d-2=3$$

$$d=5$$

$$a \cdot b \cdot c \cdot d = 4 \cdot 3 \cdot 2 \cdot 5 = 120$$

Yanıt / Answer E

19.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = ?$$

$$A) \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$B) \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C) \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$

$$D) \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$$E) \begin{bmatrix} 4 & 1 \\ 0 & 1 \end{bmatrix}$$

(YÖS 2005)

Çözüm / Solution:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

Yanıt / Answer D

$$20. A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{vmatrix} a \cdot k & b \cdot k \\ c \cdot t & d \cdot t \end{vmatrix} = ?$$

$$A) k \cdot t \cdot |A|$$

$$B) 4 \cdot k \cdot t \cdot |A|$$

$$C) k^2 \cdot t^2 \cdot |A|$$

$$D) 2 \cdot k^2 \cdot t^2 \cdot |A|$$

$$E) 2 \cdot (k+t) \cdot |A|$$

(YÖS 2005)

Çözüm / Solution:

$$\begin{vmatrix} ak & bk \\ ct & dt \end{vmatrix} = kt \begin{vmatrix} a & b \\ c & d \end{vmatrix} = k \cdot t \cdot |A|$$

Yanıt / Answer A

$$21. A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 - 5A - 2I = ?$$

$$A) \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$B) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C) \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D) \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$E) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(YÖS 2006)

Çözüm / Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Yanıt / Answer C

22. $A = \begin{bmatrix} -5 & 1 \\ 3 & -1 \end{bmatrix}$

$A^{-1} = ?$

A) $\begin{bmatrix} \frac{-1}{2} & \frac{-1}{2} \\ \frac{-3}{2} & \frac{-5}{2} \end{bmatrix}$

B) $\begin{bmatrix} -1 & -1 \\ -3 & -5 \end{bmatrix}$

C) $\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ -3 & \frac{5}{2} \end{bmatrix}$

D) $\begin{bmatrix} -1 & 2 \\ -3 & -5 \end{bmatrix}$

E) $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

(YÖS 2006)

Çözüm / Solution:

$$A = \begin{bmatrix} -5 & 1 \\ 3 & -1 \end{bmatrix} \Rightarrow \det(A) = (-5)(-1) - (3 \cdot 1) = 2$$

$$\Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & -1 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{-1}{2} \\ \frac{-3}{2} & \frac{-5}{2} \end{bmatrix}$$

Yanıt / Answer A

23. $\begin{vmatrix} \sin x & \cos x \\ \sin x & \cos x \end{vmatrix} = ?$

A) 0

B) 1

C) $\sin 2x$

D) $\sin^2 x$

E) $\sin^2 x - \cos^2 x$

(YÖS 2006)

Çözüm / Solution:

$$\begin{vmatrix} \sin x & \cos x \\ \sin x & \cos x \end{vmatrix} = \sin x \cdot \cos x - \sin x \cdot \cos x = 0$$

Yanıt / Answer A

24.

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 4 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 0 & 5 \\ 4 & 1 & -1 \end{bmatrix}$$

$\Rightarrow A \cdot B = ?$

A) $\begin{bmatrix} 6 & 3 & 7 \\ 1 & 1 & 4 \\ 12 & 6 & 14 \end{bmatrix}$

B) $\begin{bmatrix} -6 & 3 & 7 \\ 1 & -11 & 4 \\ 12 & 6 & -14 \end{bmatrix}$

C) $\begin{bmatrix} -1 & 2 & 3 \\ 1 & 1 & 4 \\ -2 & 4 & 24 \end{bmatrix}$

D) $\begin{bmatrix} -14 & 5 \\ 21 & -7 \end{bmatrix}$

E) $\begin{bmatrix} 14 & 5 \\ 21 & 7 \end{bmatrix}$

(YÖS 2007)

Çözüm / Solution:

$$A \cdot B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 & 5 \\ 4 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 7 \\ 1 & 1 & 4 \\ 12 & 6 & 14 \end{bmatrix}$$

Yanıt / Answer A

25. $\begin{vmatrix} 108765 & 108766 \\ 108763 & 108764 \end{vmatrix} = ?$

A) 1

B) 2

C) 3

D) 4

E) 6

(YÖS 2007)

Çözüm / Solution:

$108765 = x$

$$\begin{vmatrix} x & x+1 \\ x-2 & x-1 \end{vmatrix} = x^2 - x - (x^2 - x - 2)$$

$$= x^2 - x - x^2 + x + 2$$

$$= 2$$

Yanıt / Answer B

1. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \Rightarrow (B \cdot A)^{-1} = ?$

Yanıt / Answer : $\begin{bmatrix} 0 & -\frac{1}{4} \\ \frac{1}{4} & 0 \end{bmatrix}$

2. $A = \begin{bmatrix} a & 3 \\ b & c \end{bmatrix}, A = A^{-1} \Rightarrow a + c = ?$

Yanıt / Answer : 0

3. $A = \begin{bmatrix} 1001 & 1000 \\ 1000 & 1001 \end{bmatrix} \Rightarrow \det A = ?$

Yanıt / Answer : 2001

4. $A = \begin{bmatrix} 9 & -9 \\ 6 & -6 \end{bmatrix} \Rightarrow A^5 = ?$

Yanıt / Answer : $3^4 A$

5. $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$
 $\Rightarrow ((A \cdot B) \cdot C)^{-1} = ?$

Yanıt / Answer : $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

6. $A = \begin{bmatrix} x-4 & 6 \\ 2 & 4-x \end{bmatrix}, B = \begin{bmatrix} x-y & 4 \\ 12 & y-2 \end{bmatrix}$

$2A = B^T \Rightarrow x \cdot y = ?$

$\begin{bmatrix} 2(x-4) & 12 \\ 4 & 2(4-x) \end{bmatrix} = \begin{bmatrix} x-y & 4 \\ 12 & y-2 \end{bmatrix}$
 $2(x-4) = x-y \Rightarrow 2x-8 = x-y \Rightarrow x-y = 2x-8 \Rightarrow -y = x-8 \Rightarrow y = 8-x$
 $4 = y-2 \Rightarrow 4 = 8-x-2 \Rightarrow 4 = 6-x \Rightarrow x = 6-4 = 2$
 $x \cdot y = 2 \cdot (8-2) = 2 \cdot 6 = 12$

Yanıt / Answer : 12

7. $A = \begin{bmatrix} -2 & x \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} x & -6 \\ 2 & 4 \end{bmatrix}, C = \begin{bmatrix} y-4 & -8 \\ -7 & -2 \end{bmatrix}$

$A \cdot B = C \Rightarrow y - x = ?$

Yanıt / Answer : 9

8. $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow A^6 = ?$

Yanıt / Answer : $\begin{bmatrix} 64 & 0 \\ 0 & 729 \end{bmatrix}$

9. $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & -2 \\ 2 & 5 & 3 \end{bmatrix} \Rightarrow A^t \cdot B = ?$

Yanıt / Answer : $\begin{bmatrix} 6 & 17 & 5 \\ 0 & -3 & 6 \\ 8 & 21 & 10 \end{bmatrix}$

10. $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \Rightarrow A^9 = ?$

Yanıt / Answer : $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

11. $A = \begin{bmatrix} 1 & -3 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 4 & -3 \\ 1 & -1 \end{bmatrix} \Rightarrow A \cdot B^3 = ?$

Yanıt / Answer : $\begin{bmatrix} 13 & -9 \\ 3 & -2 \end{bmatrix}$

12. $\begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 3 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -5 & -4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 3 \\ 2 & -1 & 3 \end{bmatrix} = ?$

Yanıt / Answer : $\begin{bmatrix} 20 & 0 & 15 \\ -4 & 2 & -6 \end{bmatrix}$

13. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ -20 \\ 34 \end{bmatrix} \Rightarrow x+y+z = ?$

Yanıt / Answer : 22

14. $\begin{bmatrix} -2 & 2 \\ 4 & 5 \end{bmatrix} - 3 \cdot A = \begin{bmatrix} 4 & 8 \\ -5 & -7 \end{bmatrix} \Rightarrow A = ?$

Yanıt / Answer : $\begin{bmatrix} -2 & -2 \\ 3 & 4 \end{bmatrix}$

15. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^{27} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow a+b+c+d = ?$

Yanıt / Answer : 29

16. $A = \begin{bmatrix} 1 & 2 \\ 6 & 13 \end{bmatrix} \Rightarrow A^{-1} = ?$

Yanıt / Answer : $\begin{bmatrix} 13 & -2 \\ -6 & 1 \end{bmatrix}$

1. $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

$A + B = ?$

A) $\begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 0 & 4 \\ 1 & -3 & 8 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 8 \end{bmatrix}$ D) $\begin{bmatrix} 0 & -4 & 3 \\ -1 & -9 & 16 \end{bmatrix}$

E) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

2. $A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ -3 & 4 & -1 \end{bmatrix} \Rightarrow 2A = ?$ 6 8 10
2 4 6
-6 8 -2

A) $\begin{bmatrix} 6 & 8 & 5 \\ 1 & 2 & 3 \\ -6 & 8 & -2 \end{bmatrix}$ B) $\begin{bmatrix} 6 & 8 & 10 \\ 2 & 4 & 6 \\ -3 & 4 & -2 \end{bmatrix}$

C) $\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ -6 & 8 & -2 \end{bmatrix}$ D) $\begin{bmatrix} 6 & 8 & 10 \\ 2 & 4 & 6 \\ -6 & 8 & -2 \end{bmatrix}$

E) $\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 6 & 8 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 0 & 2 \end{bmatrix} = 4x \Rightarrow x = ?$

A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ C) $2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

D) $2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 2 & -3 \\ 4 & 0 \end{bmatrix} - x = \begin{bmatrix} -3 & 4 \\ 5 & -1 \end{bmatrix} \Rightarrow x = ?$ -5 7
-1 1

A) $\begin{bmatrix} 5 & -7 \\ -1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -1 & 1 \\ 9 & -1 \end{bmatrix}$ C) $\begin{bmatrix} 5 & 7 \\ -1 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 5 & -7 \\ -1 & -1 \end{bmatrix}$ E) $\begin{bmatrix} -5 & 7 \\ -1 & 1 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \end{bmatrix} \Rightarrow A^T = ?$

A) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 3 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 2 & 3 \end{bmatrix}$ C) $\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 2 & 3 \end{bmatrix}$

6. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 - 5A + 7I = x \Rightarrow x = ?$

A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ E) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

7. $A = \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A + x = B^{-1} \cdot x \Rightarrow x = ?$

A) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ C) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

D) $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ E) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

8. $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 0 \\ -1 & 2 & 0 \end{bmatrix} \Rightarrow \det A = ?$

A) -20 B) 20 C) 12 D) 10 E) 8

9. $\begin{vmatrix} x & y \\ z & t \end{vmatrix} = 5$, $\begin{vmatrix} x & y+m \\ z & t+n \end{vmatrix} = 7 \Rightarrow \begin{vmatrix} x & m \\ z & n \end{vmatrix} = ?$

A) 2 B) 5 C) 7 D) 12 E) 35

10. $\begin{vmatrix} \cos 2x & \sin 2x \\ \sin x & \cos x \end{vmatrix} = -\frac{1}{2} \Rightarrow x = ?$

A) 30° B) 40° C) 60° D) 120° E) 150°

11. $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow$

$d = ?$

A) 1 B) -3 C) -5 D) -1 E) 3

12. $A = \begin{bmatrix} a-b & 1 \\ 3 & b-2 \end{bmatrix}$, $B = \begin{bmatrix} a-6 & 6 \\ 2 & 4-a \end{bmatrix}$

$2A = B^T \Rightarrow a + 2b = ?$

A) 8 B) 6 C) 4 D) 2 E) 1

13. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} a & 2 \\ 3 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$

$a + b = ?$

A) -1 B) -5 C) -6 D) -7 E) 5

14. $\begin{vmatrix} 2 & 2 & a-1 \\ 4 & 0 & 0 \\ 1 & -1 & 4 \end{vmatrix} = 8 \Rightarrow a = ?$

A) 10 B) 11 C) 18 D) 20 E) 22

15. $A = \begin{bmatrix} 3 & 2 & 3 \\ 1 & -2 & -1 \\ -2 & 4 & 2 \end{bmatrix} \Rightarrow \det A = ?$

A) -1 B) 0 C) 1 D) 2 E) 4

Yanıtlar / Answers

1. B	2. D	3. B	4. A	5. A	6. C
7. E	8. B	9. A	10. A	11. D	12. A
13. C	14. B	15. B			

1. $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ x & y \end{bmatrix} \Rightarrow x+y = ?$

- A) -8 B) $-\frac{15}{2}$ C) -7 D) $-\frac{13}{2}$ E) -6

2. $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$, $B = [1-1 \ 3] \Rightarrow A \cdot B^T = ?$

- A) $\begin{bmatrix} 10 \\ -1 \end{bmatrix}$ B) $\begin{bmatrix} 10 \\ -2 \end{bmatrix}$ C) $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$
D) $\begin{bmatrix} -10 \\ 2 \end{bmatrix}$ E) $\begin{bmatrix} -10 \\ -2 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$,

$C = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 3 \end{bmatrix} \Rightarrow A \cdot B + C = ?$

- A) $\begin{bmatrix} 8 & 10 & 14 \\ 5 & 20 & 19 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 8 & 14 \\ 8 & 20 & 29 \end{bmatrix}$
C) $\begin{bmatrix} 5 & 10 & 14 \\ 8 & 20 & 29 \end{bmatrix}$ D) $\begin{bmatrix} 8 & 20 & 29 \\ 5 & 10 & 14 \end{bmatrix}$
E) $\begin{bmatrix} 5 & 14 & 10 \\ 8 & 20 & 29 \end{bmatrix}$

4. $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \Rightarrow A^{-1} = ?$

- A) $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$
D) $\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$ E) $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$

5. $\begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = ?$

- A) $\begin{bmatrix} -40 \\ -8 \end{bmatrix}$ B) $\begin{bmatrix} 40 \\ 8 \end{bmatrix}$ C) $\begin{bmatrix} -8 \\ -40 \end{bmatrix}$
D) $\begin{bmatrix} -8 \\ 40 \end{bmatrix}$ E) $\begin{bmatrix} 8 \\ 40 \end{bmatrix}$

6. $\begin{bmatrix} 2 & 1 \\ 1 & 8 \\ 2 & 1 \end{bmatrix}^T \cdot \begin{bmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} = ?$

- A) $\begin{bmatrix} 4 & 4 \\ 5 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 4 \\ 5 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 5 & 5 \\ 4 & 4 \end{bmatrix}$
D) $\begin{bmatrix} 5 & 4 \\ 9 & 9 \end{bmatrix}$ E) $\begin{bmatrix} 5 & 5 \\ 10 & 10 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & y \\ z & t \end{bmatrix} \Rightarrow \begin{bmatrix} x & y \\ z & t \end{bmatrix} = ?$

- A) $\begin{bmatrix} 2 & 3 \\ -2 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 3 \\ -2 & -2 \end{bmatrix}$
D) $\begin{bmatrix} -2 & 3 \\ 2 & 2 \end{bmatrix}$ E) $\begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix}$

8. $A = \begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$, $A^T \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow B = ?$

- A) $\begin{bmatrix} 5 & -6 \\ -4 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix}$ C) $\begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$
D) $\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$ E) $\begin{bmatrix} -5 & -6 \\ -4 & 5 \end{bmatrix}$

9. $A = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \Rightarrow A^2 B = ?$

A) $\begin{bmatrix} 9 & -18 \\ 14 & 76 \end{bmatrix}$ B) $\begin{bmatrix} 9 & -18 \\ -14 & 76 \end{bmatrix}$ C) $\begin{bmatrix} 9 & 14 \\ -18 & 76 \end{bmatrix}$

D) $\begin{bmatrix} 9 & 76 \\ -18 & 14 \end{bmatrix}$ E) $\begin{bmatrix} 14 & 76 \\ 9 & -18 \end{bmatrix}$

10. $A = \begin{bmatrix} \tan \alpha & 0 \\ 0 & -6 \end{bmatrix} \Rightarrow A^{-1} = ?$

A) $\begin{bmatrix} \tan \alpha & 0 \\ 0 & \sin \alpha \end{bmatrix}$ B) $\begin{bmatrix} \cot \alpha & 0 \\ 0 & -\frac{1}{6} \end{bmatrix}$

C) $\begin{bmatrix} \cot \alpha & 0 \\ 0 & -6 \end{bmatrix}$ D) $\begin{bmatrix} \sin \alpha & 0 \\ 0 & 6 \end{bmatrix}$

E) $\begin{bmatrix} \tan \alpha & 0 \\ 0 & \sec \alpha \end{bmatrix}$

11. $\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}^{1998} = ?$

A) $3^{999} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ B) $9^{999} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C) $3^{999} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ D) $9^{1998} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

E) $3^{999} \begin{bmatrix} 1998 & 0 \\ 0 & 1998 \end{bmatrix}$

12. $A = \begin{bmatrix} 1 & 3 & 5 \\ 5 & x & -3x \\ -y & 10 & 14 \end{bmatrix}$

$a_{22} + a_{23} = a_{31} \Rightarrow y = ?$

A) $-2x$ B) $-x$ C) 0 D) x E) $2x$

13. $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{18} = ?$

A) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & -9 \\ 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & -32 \\ 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -54 \\ 0 & 1 \end{bmatrix}$

E) $\begin{bmatrix} 1 & -72 \\ 0 & 1 \end{bmatrix}$

14. $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix} \Rightarrow A \cdot A^T = ?$

A) $\begin{bmatrix} 5 & 1 \\ 5 & 26 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 1 \\ 26 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 5 & 1 \\ 1 & 26 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 5 \\ 1 & 26 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 5 \\ 5 & 26 \end{bmatrix}$

15. $A - 3 \cdot \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} + 4 = 0 \Rightarrow A = ?$

A) $\begin{bmatrix} 16 & 6 \\ -3 & 13 \end{bmatrix}$ B) $\begin{bmatrix} 8 & 6 \\ -3 & 5 \end{bmatrix}$ C) $\begin{bmatrix} -3 & 6 \\ 13 & 16 \end{bmatrix}$

D) $\begin{bmatrix} 13 & 16 \\ -3 & 6 \end{bmatrix}$ E) $\begin{bmatrix} -3 & 16 \\ 13 & 6 \end{bmatrix}$

Yanitlar / Answers

1. B	2. A	3. C	4. E	5. D	6. E
7. A	8. A	9. B	10. B	11. B	12. E
13. D	14. C	15. B			

1. $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow 2A + B = ?$

A) $\begin{bmatrix} 3 & 6 \\ 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 6 \\ -1 & 3 \end{bmatrix}$ C) $\begin{bmatrix} 3 & 6 \\ 3 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x & \frac{3}{2} \\ y & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow x + y = ?$

A) 0 B) -1 C) -2 D) -3 E) -4

3. $A = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$, $A \cdot B = \begin{bmatrix} 4 & -5 \\ -10 & 9 \end{bmatrix} \Rightarrow B = ?$

A) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$ B) $\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$ C) $\begin{bmatrix} -3 & 1 \\ 2 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ E) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

4. $\begin{bmatrix} a & x \\ -x & 3 \end{bmatrix}^T \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix} \Rightarrow a = ?$

A) -4 B) -3 C) -1 D) 2 E) 3

5. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $A \cdot X = B \Rightarrow X = ?$

A) $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ E) $\begin{bmatrix} -1 & 1 \\ 1 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 5 \end{bmatrix} = \begin{bmatrix} a & \cdot & \cdot \\ \cdot & b & \cdot \\ \cdot & \cdot & c \end{bmatrix}$

$\Rightarrow a + b + c = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

7. $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$, $(A^T)^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow a + b = ?$

A) -2 B) -1 C) 4 D) 5 E) 8

8. $A = \begin{bmatrix} -\sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \Rightarrow A^{-1} = ?$

A) $\begin{bmatrix} -\sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$ B) $\begin{bmatrix} -\sin \alpha & -\cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$

C) $\begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & -\sin \alpha \end{bmatrix}$ D) $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

E) $\begin{bmatrix} -\cos \alpha & \sin \alpha \\ -\sin \alpha & -\cos \alpha \end{bmatrix}$

9. $\begin{vmatrix} a & 2 & -1 \\ 3 & -2 & 4 \\ 5 & 0 & 1 \end{vmatrix} = 46 \Rightarrow a = ?$

- A) -11 B) -1 C) 1 D) 2 E) 12

10. $A = \begin{bmatrix} 13 & 6 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix} \Rightarrow \det(A^{-1} \cdot B) = ?$

- A) 1 B) 13 C) 14 D) 26 E) 52

11. $x = \begin{vmatrix} 1 & 3 & a \\ 2 & 9 & b \\ 3 & 5 & c \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 3 & a+3 \\ 2 & 9 & b+9 \\ 3 & 5 & c+5 \end{vmatrix} = ?$

- A) $x-3$ B) $x-2$ C) $x-1$ D) x E) $x+1$

12. $\begin{vmatrix} x & 1 & x \\ 2 & 3 & 4 \\ x & 5 & x \end{vmatrix} = 24 \Rightarrow x = ?$

- A) 0 B) -1 C) -2 D) -3 E) -4

13. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

14. $K = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, K \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$K \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow K \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = ?$

A) $\begin{bmatrix} -7 \\ -6 \end{bmatrix}$ B) $\begin{bmatrix} -4 \\ -7 \end{bmatrix}$ C) $\begin{bmatrix} -7 \\ -4 \end{bmatrix}$

D) $\begin{bmatrix} -4 \\ 7 \end{bmatrix}$ E) $\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

15. $\begin{vmatrix} 2004 & 2003 \\ 2001 & 2002 \end{vmatrix} = ?$

- A) 2225 B) 3205 C) 3605 D) 3999 E) 4005

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16. $\begin{vmatrix} 2 & -\frac{1}{2} & x \\ x & 3 & 0 \\ x & 2 & 4 \end{vmatrix} = 0 \Rightarrow SS = ?$

- A) $\{-2, -4\}$ B) $\{-4, -6\}$ C) $\{-4, 6\}$
D) $\{4, 6\}$ E) $\{2, 6\}$

Yanıtlar / Answers

1. C	2. B	3. E	4. A	5. C	6. E
7. D	8. A	9. A	10. B	11. D	12. D
13. E	14. C	15. E	16. C		

1. $A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 5 \\ 0 & 1 & -4 \end{bmatrix}$

$\Rightarrow A + B = ?$

A) $\begin{bmatrix} 3 & 1 & -9 \\ 1 & 1 & 7 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 5 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ C) $\begin{bmatrix} -2 & 6 & -20 \\ 0 & 2 & -12 \end{bmatrix}$

D) $\begin{bmatrix} -2 & -\frac{3}{2} & 1 \\ 1 & 2 & -\frac{4}{3} \end{bmatrix}$ E) $\begin{bmatrix} 1 & 4 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 3x-1 \\ -4 & 5 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ 1 & 4-y \end{bmatrix}, C = \begin{bmatrix} 6 & 2 \\ z+2 & 3 \end{bmatrix}$

$A + B = C \Rightarrow x + y + z = ?$

A) 10 B) 9 C) 8 D) 5 E) 4

3. $A = \begin{bmatrix} 2 & 1 & -2 \\ 3 & -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$

$\Rightarrow A \cdot B = ?$

A) $\begin{bmatrix} -2 & -5 & 3 \\ 13 & 9 & 23 \end{bmatrix}$ B) $\begin{bmatrix} -2 & 13 \\ -5 & 9 \\ 5 & 23 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 5 & 6 \\ 9 & 7 & 12 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 4 & 5 \\ 7 & 6 & -3 \end{bmatrix}$ E) $\begin{bmatrix} -2 & 5 & -5 \\ 9 & -13 & 20 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ -3 & 1 \\ 4 & -2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 3x-6 & 4-2y \\ z-2 & 2t-4 \end{bmatrix} \Rightarrow x + y + t - z = ?$

A) 16 B) 18 C) 20 D) 22 E) 24

5. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 3 & 4 \end{bmatrix}$

$\Rightarrow (A \cdot B)^t = ?$

A) $\begin{bmatrix} 2 & 3 & 3 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 1 & 2 \\ 3 & 0 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 4 & 4 \\ 1 & 0 & 3 \\ 3 & 0 & 6 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 6 & 6 \\ 0 & 3 & 0 \\ 3 & 0 & 6 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 0 & 3 \\ 6 & 3 & 0 \\ 6 & 0 & 6 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{21} = ?$

A) $2^{20} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ B) $-2^{20} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 18 \\ 0 & 1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 21 \\ 0 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 1 \\ 0 & 21 \end{bmatrix}$

7. $A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{19} = ?$

A) $19 \cdot \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

B) $19 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D) $2^{18} \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

E) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^5 = ?$

A) $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$

D) $\begin{bmatrix} 21 & 13 \\ 13 & 8 \end{bmatrix}$

E) $\begin{bmatrix} 21 & 8 \\ 8 & 21 \end{bmatrix}$

9. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{17} = ?$

A) $2^{12} \cdot A$

B) $-2^{12} \cdot A$

C) $2^{16} \cdot A$

D) $-2^{16} \cdot A$

E) $2^{14} \cdot A$

10. $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \Rightarrow A^{-1} = ?$

A) $\frac{1}{2} \begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix}$

B) $\frac{1}{4} \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

C) $\frac{1}{4} \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

E) $\frac{1}{8} \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

11. $A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix} \Rightarrow (A \cdot B)^{12} = ?$

A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B) $\frac{1}{64} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C) $2^8 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

D) $2^{12} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

E) $2^{16} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

12. $A = \begin{bmatrix} a & \frac{1}{3} \\ \frac{1}{3} & b \end{bmatrix} A = A^{-1} \Rightarrow a - b = ?$

A) 0

B) 1

C) $\frac{2\sqrt{2}}{3}$

D) $-\frac{2\sqrt{2}}{3}$

E) $\frac{4\sqrt{2}}{3}$

13. $x + y = 9$ ve (and) $\begin{bmatrix} 2 & 3 & -4 \\ 3 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \Rightarrow z = ?$

A) $\frac{11}{2}$

B) $-\frac{9}{2}$

C) -4

D) $\frac{5}{2}$

E) 5

Yanıtlar / Answers

1. B	2. D	3. A	4. C	5. E	6. D	7. E	8. B
9. C	10. C	11. D	12. E	13. A			

$A^2 A^2$

1. $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \Rightarrow A^4 = ?$

A) $\begin{bmatrix} -5 & -24 \\ 24 & -5 \end{bmatrix}$ B) $\begin{bmatrix} -7 & -24 \\ 24 & -7 \end{bmatrix}$ C) $\begin{bmatrix} -6 & 18 \\ 18 & -6 \end{bmatrix}$

D) $32 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E) $64 \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$
 $= \begin{bmatrix} 9-16 & -12-12 \\ 16-12 & 12-9 \end{bmatrix} = \begin{bmatrix} -7 & -24 \\ 4 & 3 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow A^2 \cdot A^t = ?$

A) $\begin{bmatrix} 11 & -2 \\ 12 & 29 \end{bmatrix}$ B) $\begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix}$ C) $\begin{bmatrix} -11 & 12 \\ 2 & -29 \end{bmatrix}$

D) $\begin{bmatrix} 12 & 13 \\ 4 & 27 \end{bmatrix}$ E) $\begin{bmatrix} 11 & 2 \\ -12 & 29 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow (A \cdot B)^t = ?$

A) $\begin{bmatrix} 4 & -5 \\ -2 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 4 & 5 \\ -2 & 5 \end{bmatrix}$ C) $\begin{bmatrix} 4 & -2 \\ 5 & 5 \end{bmatrix}$

D) $\begin{bmatrix} 6 & 7 \\ -3 & 7 \end{bmatrix}$ E) $\begin{bmatrix} 4 & 2 \\ 5 & 5 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ -1 & 2 & 1 \end{bmatrix}$

$\Rightarrow (A \cdot B)^t = ?$

A) $\begin{bmatrix} 5 & -6 \\ 3 & 11 \\ -4 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 6 & 2 & -3 \\ 4 & 9 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 9 & 2 & -1 \\ -6 & 4 & -3 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 3 & -4 \\ -6 & 11 & -1 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 2 & -4 \\ -5 & 10 & -2 \end{bmatrix}$

5. $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & -2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} x-3 & y-2 \\ z-5 & t-10 \end{bmatrix} \Rightarrow x+y+t+z = ?$

A) 12 B) 13 C) 14 D) 16 E) 17

6. $A = \begin{bmatrix} 2 & -3 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$

$C = 3A - 2B \Rightarrow C = ?$

A) $\begin{bmatrix} 4 & -7 & -1 \\ -4 & -3 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 4 & -11 & 1 \\ 4 & 3 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 7 & 1 \\ 4 & 3 & 1 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 1 & 8 \\ 3 & 2 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 2 & 5 \\ 7 & 9 & 11 \end{bmatrix}$

7. $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix} - 2A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & -1 & 2 \\ 1 & -2 & 1 \end{bmatrix} \Rightarrow A = ?$

A) $\begin{bmatrix} 0 & -2 & 1 \\ -2 & 2 & 0 \\ 0 & 2 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 2 & -1 \\ -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 2 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 0 & -2 & 0 \\ -2 & 2 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & -2 & -1 \\ -1 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}$

8. $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow (A \cdot B)^t = ?$

A) $\begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 3 & 1 \\ 6 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$ E) $\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$

9. $A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & -1 & 1 \\ 1 & 3 & -2 \end{bmatrix} \Rightarrow |A| = ?$

A) 12 B) 13 C) 14 D) 16 E) 18

10. $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ -2 & 5 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 0 & 1 \\ -2 & 1 & 5 \end{bmatrix} \Rightarrow \det(A \cdot B) = ?$

A) -837 B) -927 C) -1096 D) -1279 E) -1377

11. $A = \begin{bmatrix} -2 & 5 \\ 2 & 3 \end{bmatrix}$

$B = A^t - 2A \Rightarrow B = ?$

A) $\begin{bmatrix} 2 & -8 \\ 1 & -3 \end{bmatrix}$

B) $\begin{bmatrix} -2 & 8 \\ -1 & 8 \end{bmatrix}$

C) $\begin{bmatrix} -2 & 8 \\ -1 & 3 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

E) $\begin{bmatrix} -2 & 6 \\ 3 & 8 \end{bmatrix}$

12. $A = \begin{bmatrix} 2 & 2 \\ 3 & -1 \end{bmatrix} \Rightarrow A^3 = ?$

A) $\begin{bmatrix} 10 & 2 \\ 3 & 7 \end{bmatrix}$

B) $2^3 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 26 & 18 \\ 27 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 26 & 8 \\ 21 & -1 \end{bmatrix}$

E) $\begin{bmatrix} 18 & 12 \\ 15 & -3 \end{bmatrix}$

13. $A = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$

$f(x) = 2x^2 - 4x + 3 \Rightarrow f(A) = ?$

A) $\begin{bmatrix} 18 & 13 \\ 10 & 29 \end{bmatrix}$

B) $\begin{bmatrix} 31 & 12 \\ 20 & 39 \end{bmatrix}$

C) $\begin{bmatrix} 14 & 8 \\ 12 & 16 \end{bmatrix}$

D) $\begin{bmatrix} 30 & 17 \\ 10 & 13 \end{bmatrix}$

E) $2 \begin{bmatrix} 31 & 12 \\ 20 & 39 \end{bmatrix}$

Yanıtlar / Answers

1. B	2. A	3. C	4. A	5. E	6. A
7. A	8. A	9. C	10. E	11. A	12. C
13. B					

Tanım : x değeri a 'ya yaklaşıırken $f(x)$ fonksiyonu L değerine yaklaşıyorsa $f(x)$ fonksiyonunun limiti L 'dir denir ve aşağıdaki şekilde gösterilir.

$$\lim_{x \rightarrow a} f(x) = L$$

Definition : If $f(x)$ approaches to L as x approaches to a then the limit of $f(x)$ is L and it is shown in the form below.

$$\lim_{x \rightarrow a} f(x) = L$$



BAZI ÖZELLİKLER / SOME PROPERTIES

1. $\lim_{x \rightarrow a} f(x) = f(a)$

2. $\lim_{x \rightarrow x_0} k = k \quad (k \in \mathbb{R})$

3. $\lim_{x \rightarrow x_0} [f(x) \mp g(x)] = \lim_{x \rightarrow x_0} f(x) \mp \lim_{x \rightarrow x_0} g(x)$

4. $\lim_{x \rightarrow x_0} [f(x) \cdot g(x)] = \lim_{x \rightarrow x_0} f(x) \cdot \lim_{x \rightarrow x_0} g(x)$

5. $k \in \mathbb{R}, \lim_{x \rightarrow x_0} [k \cdot f(x)] = k \cdot \lim_{x \rightarrow x_0} f(x)$

6. $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow x_0} f(x)}{\lim_{x \rightarrow x_0} g(x)} \quad \left(\lim_{x \rightarrow x_0} g(x) \neq 0 \right)$

7. $\lim_{x \rightarrow x_0} [f(x)]^n = \left[\lim_{x \rightarrow x_0} f(x) \right]^n$

8. $\lim_{x \rightarrow x_0} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow x_0} f(x)}$

$f(x) > 0$ ve n tek doğal sayı

$f(x) > 0$ and n is an odd natural number

9. $\lim_{x \rightarrow x_0} [C \cdot f(x)] = C \cdot \lim_{x \rightarrow x_0} f(x), \quad (C \in \mathbb{R})$

10. $\lim_{x \rightarrow x_0} [\log_a f(x)] = \log_a \left[\lim_{x \rightarrow x_0} f(x) \right]$

Örnek / Example:

$$f(x) = x^3 + 2x^2 - 3x + 2 \Rightarrow \lim_{x \rightarrow 2} f(x) = ?$$

Çözüm / Solution:

$$\lim_{x \rightarrow 2} f(x) = f(2) = 2^3 + 2 \cdot 2^2 - 3 \cdot 2 + 2$$

$$\lim_{x \rightarrow 2} f(x) = f(2) = 8 + 8 - 6 + 2 = 12$$

Örnek / Example:

$$f(x) = \frac{x^3 + x + 3}{x^2 + 2} \Rightarrow \lim_{x \rightarrow 3} f(x) = ?$$

Çözüm / Solution:

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{3^3 + 3 + 3}{3^2 + 2} = \lim_{x \rightarrow 3} \frac{33}{11} = 3$$



BELİRSİZLİKLER / UNCERTAINTIES

$\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 0^0 , ∞^0 , 1^∞ türündeki ifadeler belirsizlik denir.

The expressions similar to $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 0^0 , ∞^0 , 1^∞ are called uncertainties.

$$I. \frac{0}{0} \rightarrow \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$$

Bu tip belirsizliklerde limiti bulmak için $f(x)$ ve $g(x)$ çarpanlarına ayrılıp, bazı terimler sadeleştirilir.

To calculate such types of limits, we factorise $f(x)$ and $g(x)$, then simplify some of the terms.

Örnek / Example:

$$f(x) = \frac{x^2 - 1}{x^3 - 1} \Rightarrow \lim_{x \rightarrow 1} f(x) = ?$$

Çözüm / Solution:

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1} = \frac{\lim_{x \rightarrow 1} (x^2 - 1)}{\lim_{x \rightarrow 1} (x^3 - 1)} = \frac{1 - 1}{1 - 1} = \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{(x - 1)(x^2 + x + 1)} \\ &= \lim_{x \rightarrow 1} \frac{x + 1}{x^2 + x + 1} = \frac{1 + 1}{1 + 1 + 1} = \frac{2}{3} \end{aligned}$$

Örnek / Example:

$$f(x) = \frac{\sqrt{x} - 2}{x^3 - 64} \Rightarrow \lim_{x \rightarrow 4} f(x) = ?$$

Çözüm / Solution:

$$\lim_{x \rightarrow 4} f(x) = \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^3 - 64} = \frac{\sqrt{4} - 2}{4^3 - 64} = \frac{2 - 2}{64 - 64} = \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 4} f(x) &= \lim_{x \rightarrow 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(x - 4)(x^2 + 4x + 16)(\sqrt{x} + 2)} \\ &= \lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(x^2 + 4x + 16)(\sqrt{x} + 2)} \\ &= \lim_{x \rightarrow 4} \frac{1}{(x^2 + 4x + 16)(\sqrt{x} + 2)} \\ &= \frac{1}{(16 + 16 + 16) \cdot (2 + 2)} = \frac{1}{48 \cdot 4} = \frac{1}{192} \end{aligned}$$

$$II. \frac{\infty}{\infty} \rightarrow \lim_{x \rightarrow \pm \infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}$$

$$n > m \Rightarrow \text{limit} = \pm \infty$$

$$n = m \Rightarrow \text{limit} = \frac{a_n}{b_m}$$

$$n < m \Rightarrow \text{limit} = 0$$

Örnek / Example:

$$f(x) = \frac{3x^2 + 4x - 5}{6x^2 + x + 3} \Rightarrow \lim_{x \rightarrow \infty} f(x) = ?$$

Çözüm / Solution:

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{3x^2 + 4x - 5}{6x^2 + x + 3} = \frac{\infty + \infty - 5}{\infty + \infty + 3} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2 \left(3 + \frac{4}{x} - \frac{5}{x^2} \right)}{x^2 \left(6 + \frac{1}{x} + \frac{3}{x^2} \right)}$$

$$\lim_{x \rightarrow \infty} \frac{3 + 0 - 0}{6 + 0 + 0} = \frac{3}{6} = \frac{1}{2}$$

III. $\infty - \infty$, $0 \cdot \infty$ türündeki belirsizlikler, $\frac{\infty}{\infty}$ ya da $\frac{0}{0}$ türüne çevrilir.

To solve this types of uncertainties, similar to $\infty - \infty$ and $0 \cdot \infty$, we convert them into the type $\frac{\infty}{\infty}$ or $\frac{0}{0}$ types.

Then apply the rule expressed in a and b.

Örnek / Example:

$$f(x) = 2\sqrt{x^2 + 1} - \sqrt{4x^2 + 2x + 3} \Rightarrow \lim_{x \rightarrow \infty} f(x) = ?$$

Çözüm / Solution:

$$\lim_{x \rightarrow \infty} \frac{(2\sqrt{x^2 + 1} - \sqrt{4x^2 + 2x + 3})(2\sqrt{x^2 + 1} + \sqrt{4x^2 + 2x + 3})}{2\sqrt{x^2 + 1} + \sqrt{4x^2 + 2x + 3}}$$

$$= \lim_{x \rightarrow \infty} \frac{4x^2 + 4 - 4x^2 - 2x - 3}{x(2\sqrt{1 + \frac{1}{x^2}} + \sqrt{4 + \frac{2}{x} + \frac{3}{x^2}})} = \lim_{x \rightarrow \infty} \frac{x(-2 + \frac{1}{x})}{x(2 + 2)} = -\frac{1}{2}$$

Örnek / Example:

$$f(x) = \left(1 + \frac{3x}{x^2 + 2}\right)^{2x} = ?$$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x + \frac{2}{x}}\right)^{2x} \\ &= \lim_{x \rightarrow \infty} (1 + 0)^{\infty} = 1^{\infty} \\ u(x) &= \frac{3x}{x^2 + 2} \\ \lim_{x \rightarrow \infty} u(x) &= \lim_{x \rightarrow \infty} \frac{3x}{x^2 + 2} = 0 \\ \vartheta(x) &= 2x \\ \lim_{x \rightarrow \infty} \vartheta(x) &= \infty \\ u(x) \cdot \vartheta(x) &= \frac{6x^2}{x^2 + 2} \\ \lim_{x \rightarrow \infty} [u(x) \cdot \vartheta(x)] &= \lim_{x \rightarrow \infty} \frac{6x^2}{x^2 + 2} = 6 \\ \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} \left(1 + \frac{3x}{x^2 + 2}\right)^{2x} = e^6 \end{aligned}$$

Örnek / Example:

$$f(x) = \left(e^{\frac{2}{x}} + \frac{2}{x}\right)^x \Rightarrow \lim_{x \rightarrow \infty} f(x) = ?$$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) &= \lim_{x \rightarrow \infty} \left(e^{\frac{2}{x}} + \frac{2}{x}\right)^x \\ &= \lim_{x \rightarrow \infty} \left(e^{\frac{2}{\infty}} + \frac{2}{\infty}\right)^{\infty} = (e^0 + 0)^{\infty} = 1^{\infty} \\ y &= \left(e^{\frac{2}{x}} + \frac{2}{x}\right)^x \Rightarrow \ln y = x \cdot \ln \left(e^{\frac{2}{x}} + \frac{2}{x}\right) \\ \lim_{x \rightarrow \infty} \frac{\ln \left(e^{\frac{2}{x}} + \frac{2}{x}\right)}{\frac{1}{x}} &= \lim_{x \rightarrow \infty} \frac{-\frac{2}{x^2} e^{\frac{2}{x}} - \frac{2}{x^2}}{-\frac{1}{x^2}} = 2(e^0 + 1) = 4 \\ \ln y &= 4 \Rightarrow \lim_{x \rightarrow \infty} y = e^4 \end{aligned}$$



TRİGONOMETRİK FONKSİYONLARIN LİMİTİ

LIMITS of TRIGONOMETRIC FUNCTIONS

$$f(x) = \sin x, \quad g(x) = \cos x$$

$$\begin{aligned} 1. \quad \lim_{x \rightarrow a} f(x) &= f(a) = \sin a \\ \lim_{x \rightarrow a} g(x) &= g(a) = \cos a \end{aligned}$$

$$2. \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$3. \quad \lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

$$4. \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

Örnek / Example:

$$f(x) = \frac{\sin 5x}{x} = ?$$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{5 \sin 5x}{5 \cdot x} \\ &= 5 \lim_{x \rightarrow 0} \frac{\sin 5x}{5 \cdot x} \\ \lim_{x \rightarrow 0} f(x) &= 5 \lim_{u \rightarrow 0} \frac{\sin u}{u} = 5 \cdot 1 = 5 \end{aligned}$$

Örnek / Example:

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{4x^2} = ?$$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{4x^2} &= \lim_{x \rightarrow 0} \frac{1 - \cos 0}{4 \cdot 0} = \frac{1 - 1}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{4x^2} &= \lim_{x \rightarrow 0} \frac{1 - (1 - 2 \sin^2 x)}{4 \cdot x^2} \\ \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{4x^2} &= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{4x^2} \\ &= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^2 = \frac{1}{2} \end{aligned}$$

Örnek / Example:

$$f(x) = \frac{\sin 4x \cdot \tan 2x}{1 - \cos 2x} \Rightarrow \lim_{x \rightarrow 0} f(x) = ?$$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot \tan 2x}{1 - \cos 2x} \\ &= \lim_{x \rightarrow 0} \frac{\sin 0 \cdot \tan 0}{1 - \cos 0} \\ &= \frac{0 \cdot 0}{1 - 1} = \frac{0}{0} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{2 \cdot \sin 2x \cdot \cos 2x \cdot \frac{\sin 2x}{\cos 2x}}{1 - (1 - 2 \sin^2 x)} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin^2 2x}{2 \sin^2 x} \\ &= \lim_{x \rightarrow 0} \frac{2 \cdot 4 \sin^2 x \cdot \cos^2 x}{2 \sin^2 x} \\ &= \lim_{x \rightarrow 0} 4 \cos^2 x = 4 \cdot \cos^2 0 \\ &= 4 \cdot 1 = 4 \end{aligned}$$

Örnek / Example:

$$\lim_{x \rightarrow \pi} \frac{\cos 2x - 1}{\sin^2 x} = ?$$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow \pi} \frac{\cos 2x - 1}{\sin^2 x} &= \lim_{x \rightarrow \pi} \frac{\cos 2\pi - 1}{\sin^2 \pi} \\ &= \frac{1 - 1}{0} = \frac{0}{0} \\ \lim_{x \rightarrow \pi} \frac{\cos 2x - 1}{\sin^2 x} &= \lim_{x \rightarrow \pi} \frac{1 - 2 \sin^2 x - 1}{\sin^2 x} \\ &= \lim_{x \rightarrow \pi} \frac{-2 \sin^2 x}{\sin^2 x} = -2 \end{aligned}$$

ÇÖZÜMLÜ TEST / TEST WITH SOLUTIONS

1. $\lim_{x \rightarrow -1} \frac{x^3 + 5x^2 - 2x}{3x^4 - 2x^3 - 4x^2 + 1} = ?$

- A) 1 B) 2 C) 3 D) 4 E) 5

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow (-1)} \frac{x^3 + 5x^2 - 2x}{3x^4 - 2x^3 - 4x^2 + 1} &= \frac{(-1)^3 + 5 \cdot (-1)^2 - 2 \cdot (-1)}{3 \cdot (-1)^4 - 2 \cdot (-1)^3 - 4 \cdot (-1)^2 + 1} \\ &= \frac{-1 + 5 + 2}{3 + 2 - 4 + 1} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

Yanıt / Answer C

2. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = ?$

- A) 0 B) 3 C) 5 D) 6 E) 9

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} &= \frac{0}{0} \\ \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} &= \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{x - 3} \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

Yanıt / Answer D

3. $\lim_{x \rightarrow -3} \frac{x^3 + 8}{x + 2} = ?$

- A) 15 B) 19 C) 21 D) 27 E) 30

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow -3} \frac{x^3 + 8}{x + 2} &= \frac{(-3)^3 + 8}{-3 + 2} \\ &= \frac{-27 + 8}{-1} \\ &= \frac{-19}{-1} \\ &= 19 \end{aligned}$$

Yanıt / Answer B

4. $\lim_{x \rightarrow 3} \frac{\cos x - \sin 3^\circ}{\sin x - \cos 3^\circ} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution:

$$\lim_{x \rightarrow 3} \frac{\cos x - \sin 3^\circ}{\sin x - \cos 3^\circ} = \frac{\cos 3^\circ - \sin 3^\circ}{\sin 3^\circ - \cos 3^\circ} = -1$$

Yanıt / Answer B

5. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot x - 2\cos x}{\sin x + \tan x} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cot x - 2\cos x}{\sin x + \tan x} &= \frac{\cot \frac{\pi}{6} - 2\cos \frac{\pi}{6}}{\sin \frac{\pi}{6} + \tan \frac{\pi}{6}} \\ &= \frac{\sqrt{3} - 2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{3}} \\ &= \frac{\sqrt{3} - \sqrt{3}}{\frac{3 + 2\sqrt{3}}{6}} = 0 \end{aligned}$$

Yanıt / Answer C

6. $\lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x^3 - 8} = ?$

- A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{6}$ E) $\frac{6}{5}$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x^3 - 8} &= \frac{8 - 4 - 4}{8 - 8} = \frac{0}{0} \\ \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x^3 - 8} &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 2)}{(x-2)(x^2 + 2x + 4)} \\ &= \frac{2^2 + 2 \cdot 2 + 2}{2^2 + 2 \cdot 2 + 4} \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$

Yanıt / Answer D

7. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = ?$

- A) -3 B) -1 C) 0 D) 1 E) 3

Çözüm / Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = 3 \cdot \lim_{(3x) \rightarrow 0} \frac{\sin 3x}{3x} = 3$$

Yanıt / Answer E

8. $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 - 1} = ?$

- A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 - 1} &= \frac{\sin 0}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 - 1} &= \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x-1)(x+1)} \end{aligned}$$

$$= \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1} \cdot \lim_{x \rightarrow 1} \frac{1}{x+1}$$

$$x-1=t$$

$$x \rightarrow 1 \Rightarrow t \rightarrow 0 \quad = \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \lim_{t \rightarrow 0} \frac{1}{t+2}$$

$$= 1 \cdot \frac{1}{2} = \frac{1}{2}$$

Yanıt / Answer A

9. $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 4 E) $\frac{5}{2}$

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} &= \frac{\sqrt{1} - 1}{1 - 1} = \frac{0}{0} \\ \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} &= \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{(\sqrt{x} - 1)(\sqrt{x} + 1)} \\ &= \lim_{x \rightarrow 1} \frac{1}{\sqrt{x} + 1} \\ &= \frac{1}{1+1} = \frac{1}{2} \end{aligned}$$

Yanıt / Answer B

10. $\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = ?$

- A) 3a B) a^2 C) $3a^2$ D) $5a^2$ E) $6a^2$

Çözüm / Solution:

$$\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = \frac{a^3 - a^3}{a - a} = \frac{0}{0}$$

$$\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = \lim_{x \rightarrow a} \frac{(x - a)(x^2 + ax + a^2)}{x - a}$$

$$= \lim_{x \rightarrow a} (x^2 + ax + a)$$

$$= a^2 + a \cdot a + a^2 = 3a^2$$

Yanıt / Answer C

11. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} = ?$

- A) $-\frac{5}{2}$ B) $-\frac{2}{5}$ C) $\frac{1}{5}$ D) $\frac{2}{5}$ E) $\frac{5}{2}$

Çözüm / Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{2x \cdot \frac{\sin 2x}{2x}}{5x \cdot \frac{\sin 5x}{5x}}$$

$$= \lim_{x \rightarrow 0} \frac{2x}{5x} \cdot \frac{\lim_{2x \rightarrow 0} \frac{\sin 2x}{2x}}{\lim_{5x \rightarrow 0} \frac{\sin 5x}{5x}}$$

$$= \frac{2}{5} \cdot \frac{1}{1} = \frac{2}{5}$$

Yanıt / Answer D

12. $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x - 2} = ?$

- A) -4 B) -2 C) 0 D) 2 E) 4

Çözüm / Solution:

$$\lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x - 2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{(x + 2)\sin(x^2 - 4)}{x^2 - 4} =$$

$$x^2 - 4 = t \Rightarrow \lim_{x \rightarrow 2} (x^2 - 4) = 0$$

$$= \lim_{x \rightarrow 2} (x + 2) \cdot \lim_{t \rightarrow 0} \frac{\sin t}{t}$$

$$= 4 \cdot 1 = 4$$

Yanıt / Answer E

13. $\lim_{x \rightarrow \infty} 5^{\frac{2}{x}} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution:

$$\lim_{x \rightarrow \infty} 5^{\frac{2}{x}} = 5^{\frac{2}{\infty}} = 5^0 = 1$$

Yanıt / Answer D

14. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} = ?$

- A) $\frac{5}{4}$ B) 1 C) $\frac{5}{6}$ D) $\frac{5}{8}$ E) $\frac{1}{2}$

Çözüm / Solution:

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} = \frac{4 + 2 - 6}{4 - 4} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{(x + 3)(x - 2)}{(x - 2)(x + 2)}$$

$$= \frac{2 + 3}{2 + 2}$$

$$= \frac{5}{4}$$

Yanıt / Answer A

15. $\lim_{x \rightarrow 2} \frac{\sqrt{x + 2} - 2}{x - 2} = ?$

- A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) 2

Çözüm / Solution:

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x-2} &= \frac{\sqrt{4} - 2}{2-1} = \frac{0}{0} \\&= \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x-2} \\&= \lim_{x \rightarrow 2} \frac{(\sqrt{x+2} - 2)(\sqrt{x+2} + 2)}{(x-2)(\sqrt{x+2} + 2)} \\&= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(\sqrt{x+2} + 2)} \\&= \frac{1}{\sqrt{2+2} + 2} = \frac{1}{4}\end{aligned}$$

Yanıt / Answer B

16. $\lim_{x \rightarrow \infty} \frac{x^3 - 3}{3x^3 + 2x + 1} = ?$

- A) $-\frac{1}{3}$ B) $-\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

Çözüm / Solution:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^3 - 3}{3x^3 + 2x + 1} &= \lim_{x \rightarrow \infty} \frac{x^3 \cdot \left(1 - \frac{2}{x^2} + \frac{5}{x^3}\right)}{x^3 \cdot \left(3 + \frac{1}{x^2} - \frac{2}{x^3}\right)} \\&= \frac{1 - \frac{2}{\infty^2} + \frac{5}{\infty^3}}{3 + \frac{1}{\infty^2} - \frac{2}{\infty^3}} \\&= \frac{1-0+0}{3+0-0} \\&= \frac{1}{3}\end{aligned}$$

Yanıt / Answer C

17. $\lim_{x \rightarrow \frac{\pi}{3}} \frac{3x - \pi}{\cos \frac{9x}{2}} = ?$

- A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) 1 E) $\frac{4}{3}$

Çözüm / Solution:

$$\begin{aligned}\lim_{x \rightarrow \frac{\pi}{3}} \frac{3x - \pi}{\cos \frac{9x}{2}} &= \frac{0}{0} \left(\lim_{x \rightarrow \frac{\pi}{3}} \frac{9x - 3\pi}{2} = 0 \right) \\&= \lim_{x \rightarrow \frac{\pi}{3}} \frac{3x - \pi}{\cos \frac{9x}{2}} = \frac{3x - \pi}{-\sin \left(\frac{3\pi}{2} - \frac{9\pi}{2} \right)} \\&= \lim_{x \rightarrow \frac{\pi}{3}} \frac{9x - 3\pi}{2} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{3}{2} \sin \left(\frac{9x - 3\pi}{2} \right) \\&= \frac{1}{\frac{3}{2}} \cdot 1 = \frac{2}{3}\end{aligned}$$

Yanıt / Answer C

18. $\lim_{x \rightarrow \infty} \frac{x^2 - 3}{x^3 + 2x + 1} = ?$

- A) 3 B) 2 C) 1 D) 0 E) -1

Çözüm / Solution:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x^2 - 3}{x^3 + 2x + 1} &= \lim_{x \rightarrow \infty} \frac{x^2 \cdot \left(1 - \frac{3}{x^2}\right)}{x^2 \cdot \left(x + \frac{2}{x} + \frac{1}{x^2}\right)} \\&= \frac{1 - \frac{3}{\infty^2}}{\infty + \frac{2}{\infty} + \frac{1}{\infty^2}} \\&= \frac{1-0}{\infty+0+0} \\&= \frac{1}{\infty} = 0\end{aligned}$$

Yanıt / Answer D

19. $\lim_{x \rightarrow \infty} \frac{x^5 - x^4 + 1}{x^3 + 2x} = ?$

- A) $-\infty$ B) $-\frac{5}{3}$ C) 0 D) $\frac{5}{3}$ E) ∞

Çözüm / Solution:

$$\lim_{x \rightarrow \infty} \frac{x^5 - x^4 + 1}{x^3 + 2x} = \lim_{x \rightarrow \infty} \frac{x^3 \cdot \left(x^2 - x + \frac{1}{x^3} \right)}{x^3 \cdot \left(1 + \frac{2}{x^2} \right)}$$

$$= \frac{\infty^2 - \infty + \frac{1}{\infty^3}}{1 + \frac{2}{\infty^2}} = \infty$$

Yanıt / Answer E

20. $\lim_{x \rightarrow -1} \frac{\sin \pi x}{x+1} = ?$

- A) $-\pi$ B) $-\frac{\pi}{2}$ C) 0 D) $\frac{\pi}{2}$ E) π

Çözüm / Solution:

$$\lim_{x \rightarrow -1} \frac{\sin(\pi x)}{x+1} = \frac{0}{0}$$

$$x+1 = t$$

$$x = t-1$$

$$x \rightarrow -1 \Rightarrow t \rightarrow 0$$

$$\lim_{t \rightarrow 0} \frac{-\sin(\pi - \pi t)}{t} = \lim_{t \rightarrow 0} \frac{-\pi \sin \pi t}{\pi t} = -\pi$$

Yanıt / Answer A

21. $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{1 - \sin \frac{x}{2}} = ?$

- A) 2 B) 3 C) 4 D) $\frac{3}{4}$ E) $\frac{9}{2}$

Çözüm / Solution:

$$\lim_{x \rightarrow \pi} \frac{1 + \cos x}{1 - \sin \frac{x}{2}} = \frac{1 + \cos \pi}{1 - \sin \frac{\pi}{2}} = \frac{0}{0}$$

$$\lim_{x \rightarrow \pi} \frac{1 + \cos x}{1 - \sin \frac{x}{2}} = \lim_{x \rightarrow \pi} \frac{-\sin x}{-\frac{1}{2} \cos \frac{x}{2}}$$

$$\lim_{x \rightarrow \pi} \frac{-\cos x}{\frac{1}{4} \sin \frac{x}{2}} = \frac{-\cos \pi}{\frac{1}{4} \sin \frac{\pi}{2}}$$

$$= \frac{-(-1)}{\frac{1}{4} \cdot 1} = 4$$

Yanıt / Answer C

YÖS SORULARI / YÖS QUESTIONS

1. $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^2 - x - ax + a} = ?$

- A) $2a^2$ B) $2a$ C) a D) $\frac{3}{2}a$ E) $\frac{2a}{a-1}$

(YÖS 2001)

Çözüm / Solution:

$$\lim_{x \rightarrow a} \frac{(x-a)(x+a)}{(x-a)(x-1)} = \frac{2a}{a-1}$$

Yanıt / Answer E

2. $\lim_{x \rightarrow 0} \left(\frac{\tan^2 3x}{25x^2} \right) = ?$

- A) $\frac{3}{25}$ B) $\frac{6}{25}$ C) $\frac{9}{25}$ D) 6 E) 9

(YÖS 2002)

Çözüm / Solution:

$$\lim_{x \rightarrow 0} \left[\frac{\tan 3x}{3x} \cdot \frac{\tan 3x}{3x} \cdot \frac{9}{25} \right]$$

$$= \frac{9}{25} \lim_{x \rightarrow 0} \frac{\tan 3x}{3x} \cdot \lim_{x \rightarrow 0} \frac{\tan 3x}{3x} = \frac{9}{25}$$

Yanıt / Answer C

3. $\lim_{x \rightarrow -\infty} \left(5^{\frac{3}{x}} + 4^x + 3 \right) = ?$

- A) 0 B) 1 C) 2 D) 3 E) 4

(YÖS 2002)

Çözüm / Solution:

$$\lim_{x \rightarrow -\infty} \left(5^{\frac{3}{x}} + 4^x + 3 \right) = 5^{\frac{3}{-\infty}} + 4^{-\infty} + 3$$

$$= 5^0 + \frac{1}{4^{\infty}} + 3 = 4 + \frac{1}{\infty} = 4$$

Yanıt / Answer E

4. $\lim_{x \rightarrow \frac{1}{3}} \left(\frac{x^3 - \frac{1}{27}}{x^2 - \frac{1}{9}} \right) = ?$

- A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) 2 E) 3

(YÖS 2002)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow \frac{1}{3}} \frac{\left(x - \frac{1}{3}\right)\left(x^2 + \frac{x}{3} + \frac{1}{9}\right)}{\left(x - \frac{1}{3}\right)\left(x + \frac{1}{3}\right)} \\ = \frac{\left(\frac{1}{3}\right)^2 + \frac{1}{9} + \frac{1}{9}}{\frac{1}{3} + \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \end{aligned}$$

Yanıt / Answer A

5. $\lim_{x \rightarrow \infty} \frac{x^2 - x + 3}{-x^5 + x^2} = ?$

- A) 0 B) 1 C) $\frac{1}{3}$ D) $\frac{1}{5}$ E) ∞

(YÖS 2003)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^2 - x + 3}{-x^5 + x^2} &= \frac{\infty - \infty}{-\infty + \infty} \\ \lim_{x \rightarrow \infty} \frac{x^2 - x + 3}{-x^5 + x^2} &= \lim_{x \rightarrow \infty} \frac{x^2 \left(1 - \frac{1}{x} + \frac{3}{x^2}\right)}{x^2(1 - x^3)} \\ \lim_{x \rightarrow \infty} \frac{1 - 0 + 0}{1 - \infty} &= \frac{1}{-\infty} = 0 \end{aligned}$$

Yanıt / Answer A

6. $\lim_{x \rightarrow 1} \frac{2x^3 + x^2 - 1}{x^3 + 3} = ?$

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) 2 D) 3 E) 4

(YÖS 2004)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{2x^3 + x^2 - 1}{x^3 + 3} &= \frac{2 \cdot 1^3 + 1^2 - 1}{1^3 + 3} \\ &= \frac{2}{4} = \frac{1}{2} \end{aligned}$$

Yanıt / Answer A

7. $\lim_{x \rightarrow 0} \frac{\pi \sin \frac{\pi x}{6}}{3x \cos \frac{\pi x}{3}} = ?$

- A) $\frac{\pi}{9}$ B) $\frac{\pi}{19}$ C) $\frac{\pi^2}{18}$ D) 2π E) $3\pi + 1$

(YÖS 2004)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\pi \sin \frac{\pi x}{6}}{3x \cos \frac{\pi x}{3}} &= \pi \cdot \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{6}}{3x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos \frac{\pi x}{3}} \\ &= \pi \cdot \frac{\frac{\pi}{6}}{3} \cdot 1 \\ &= \frac{\pi^2}{18} \end{aligned}$$

Yanıt / Answer C

8. $\lim_{x \rightarrow +\infty} \frac{2x^3 + 5x^2 - 7x}{3x^2 + 5} = ?$

- A) $-\infty$ B) $+\infty$ C) 0 D) 1 E) $\frac{2}{3}$

(YÖS 2005)

Çözüm / Solution:

$$\lim_{x \rightarrow +\infty} \frac{2x^3 + 5x^2 - 7x}{3x^2 + 5} = +\infty$$

Yanıt / Answer B

9. $\lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3} = ?$

- A) -1 B) 0 C) 1 D) 3 E) $+\infty$

(YÖS 2005)

Çözüm / Solution:

$$\lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^+} \frac{x-3}{x-3} = 1$$

Yanıt / Answer C

10. $\lim_{x \rightarrow 2} \frac{x^3 + 8}{4 - x^2} = ?$

- A) 0 B) 1 C) 3 D) $\frac{1}{2}$ E) $\frac{3}{2}$

(YÖS 2006)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^3 + 8}{4 - x^2} &= \lim_{x \rightarrow 2} \frac{(x+2)(x^2 - 2x + 4)}{(2-x)(2+x)} \\ &= \lim_{x \rightarrow 2} \frac{(x^2 - 2x + 4)}{2-x} \\ &= \frac{(-2)^2 - 2(-2) + 4}{2 - (-2)} = \frac{12}{4} = 3 \end{aligned}$$

Yanıt / Answer C

11. $\lim_{x \rightarrow (-3)^-} \frac{x^2 + x - 6}{|x + 3|} = ?$

- A) -5 B) -3 C) 0 D) 3 E) 5

(YÖS 2006)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow (-3)^-} \frac{x^2 + x - 6}{|x + 3|} &= \lim_{x \rightarrow (-3)^-} \frac{(x+3) \cdot (x-2)}{-x-3} \\ &= \lim_{x \rightarrow (-3)^-} -x + 2 = -(-3) + 2 = 5 \end{aligned}$$

Yanıt / Answer E

12. $\lim_{x \rightarrow a^-} \frac{|x-a|}{x-a} = ?$

- A) -1 B) 0 C) 1 D) a E) -a

(YÖS 2007)

Çözüm / Solution:

$$\lim_{x \rightarrow a^-} \frac{|x-a|}{x-a} = \lim_{x \rightarrow a^-} \frac{-(x-a)}{x-a} = -1$$

Yanıt / Answer A

13. $\lim_{x \rightarrow 0} \left[\tan \left(1 - \frac{\sin x}{x} \right) \right] = ?$

- A) $-\pi$ B) π C) 0 D) -1 E) 1

(YÖS 2007)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \left[\tan \left(1 - \frac{\sin x}{x} \right) \right] &= \tan \left(1 - \lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \\ &= \tan(1-1) = \tan 0 = 0 \end{aligned}$$

Yanıt / Answer C

14. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+27} - 3}{x} = ?$

- A) $-\frac{1}{24}$ B) $-\frac{1}{9}$ C) $-\frac{1}{3}$
D) $\frac{1}{27}$ E) $\frac{1}{3}$

(YÖS 2007)

Çözüm / Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt[3]{x+27} - 3}{x} &= \lim_{x \rightarrow 0} \frac{1}{\frac{\sqrt[3]{(x+27)^2}}{1}} = \frac{1}{3 \cdot \sqrt[3]{27^2}} = \frac{1}{27} \end{aligned}$$

Yanıt / Answer D

1. $\lim_{b \rightarrow 2} \frac{\sin(\pi \cdot b)}{4 - b^2} = ?$

Yanıt / Answer : $-\frac{\pi}{4}$

2. $\lim_{x \rightarrow 0} \frac{\ln(1 + 4x)}{\sin 5x} = ?$

Yanıt / Answer : $\frac{4}{5}$

3. $\lim_{x \rightarrow 2} \frac{\tan(2x - 4)}{x - 2} = ?$

Yanıt / Answer : 2

4. $\lim_{x \rightarrow 1} \tan\left(\frac{\pi}{2} x\right) \cdot (x - 1) = ?$

Yanıt / Answer : $\frac{2}{\pi}$

5. $\lim_{x \rightarrow 2} [(x - 2) \cdot \ln(x - 2)] = ?$

Yanıt / Answer : 0

6. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = ?$

Yanıt / Answer : 0

7. $\lim_{x \rightarrow 0} \frac{\sqrt[4]{1 + 2x} - 1}{x} = ?$

Yanıt / Answer : $\frac{1}{2}$

8. $\lim_{x \rightarrow 2} \frac{3 - \sqrt{5x - 1}}{\sqrt{x + 2} - x} = ?$

Yanıt / Answer : $\frac{10}{9}$

9. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = ?$

Yanıt / Answer : $\sqrt{2}$

10. $\lim_{\theta \rightarrow x} \frac{\tan x - \tan \theta}{\tan(\theta - x)} = ?$

Yanıt / Answer : $\sec^2 x$

11. $\lim_{x \rightarrow 3} \frac{x^3 - 3x^2 + x - 3}{x^2 - x - 6} = ?$

Yanıt / Answer : 2

12. $\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 7x} - \sqrt{x^2 + 2x} \right) = ?$

Yanıt / Answer : $\frac{5}{2}$

13. $\lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 + 6x} - \sqrt{4x^2 + 9} \right) = ?$

Yanıt / Answer : $-\frac{3}{2}$

14. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \tan x} = ?$

Yanıt / Answer : $-\frac{\sqrt{2}}{2}$

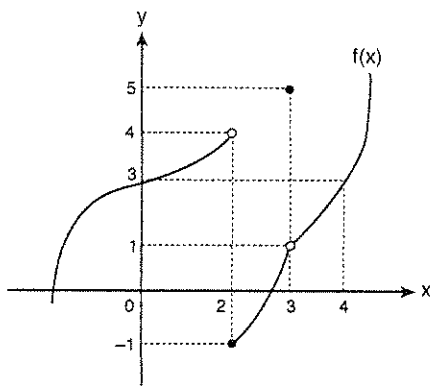
15. $\lim_{x \rightarrow 2} \frac{4x^2 - 2x + m - 2}{x^2 - 4} \in \mathbb{R} \Rightarrow m = ?$

Yanıt / Answer : $\frac{7}{2}$

16. $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{\sqrt{x+16} - 4} = ?$

Yanıt / Answer : $\frac{4}{3}$

1.



$$\Rightarrow \lim_{x \rightarrow 2^-} f(x) + \lim_{x \rightarrow 3} f(x) + \lim_{x \rightarrow 4^+} f(x) = ?$$

- A) 12 B) 10 C) 9 D) 8 E) 6

2. $\lim_{x \rightarrow 2^+} (x^2 - 4x + 1) = ?$

- A) -4 B) -3 C) -2 D) 2 E) 3

3. $f(x) = \begin{cases} x+2 & x \geq 2 \\ 2x-1 & x < 2 \end{cases}$

$$\Rightarrow \lim_{x \rightarrow 2^-} f(x) + \lim_{x \rightarrow 2^+} f(x) + \lim_{x \rightarrow 3} f(x) = ?$$

- A) 16 B) 15 C) 14 D) 13 E) 12

4. $\lim_{x \rightarrow 2^-} \left(\frac{|x-2|}{x-2} \right) + \lim_{x \rightarrow 2^+} \left(\frac{|x-2|}{x-2} \right) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

5. $\lim_{x \rightarrow 3^-} \frac{\sqrt{x^2 - 6x + 9}}{x-3} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

6. $\lim_{x \rightarrow 3} \frac{x^3 - 6x + 1}{x^2 - 1} = ?$

- A) $\frac{5}{4}$ B) $\frac{3}{2}$ C) $\frac{7}{4}$ D) 2 E) $\frac{9}{4}$

7. $\lim_{x \rightarrow y \rightarrow 0} \frac{x^2 + y^2}{x+y} = ?$

- A) -2y B) -y C) 0 D) y E) 2y

8. $\lim_{x \rightarrow e^2} (\ln x^2) = ?$

- A) 1 B) 2 C) 4 D) 8 E) 16

9. $\lim_{x \rightarrow \frac{\pi}{6}} (\sin x + \cos^2 x) = ?$

- A) $\frac{5}{4}$ B) $\frac{3}{2}$ C) $\frac{7}{4}$ D) 2 E) $\frac{9}{4}$

10. $\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x + 2} \right) = ?$

- A) -2 B) 0 C) 2 D) 4 E) 6

11. $\lim_{x \rightarrow 6} \left(\log_2 \sqrt{x^2 - 4} \right) = ?$

- A) -2 B) $\frac{9}{2}$ C) 2 D) 4 E) $\frac{5}{2}$

12. $\lim_{x \rightarrow 0} \left(\frac{\sin 3x}{x} + \frac{2x}{\tan x} \right) = ?$

- A) 5 B) 6 C) 7 D) 8 E) 10

13. $\lim_{x \rightarrow 2} \left(\frac{\sin(x-2)}{2x-4} \right) = ?$

- A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) $\frac{1}{5}$

14. $\lim_{x \rightarrow 0} \left(\frac{\sin^3 x}{8x^3} \right) = ?$

- A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) 2

15. $\lim_{x \rightarrow 2} \frac{\tan(x-2)^3}{(2x-4)^3} = ?$

- A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) 2

16. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x} = ?$

- A) 0 B) $\frac{\sqrt{2}}{2}$ C) $\frac{1}{2}$ D) $\sqrt{3}$ E) $\sqrt{2}$

Yanıtlar / Answers

1. D	2. B	3. E	4. C	5. B	6. A
7. D	8. C	9. A	10. B	11. E	12. A
13. D	14. A	15. A	16. E		

1. $\lim_{x \rightarrow 1} \frac{x^8 - 1}{x - 1} = ?$
 A) 1 B) 2 C) 4 D) 8 E) 16
2. $\lim_{x \rightarrow 9} \frac{2\sqrt{x} - 6}{x - 9} = ?$
 A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$
3. $\lim_{m \rightarrow x} \frac{m^3 - x^3}{m - x} = ?$
 A) $2x^2$ B) $3x^2$ C) $4x^2$
 D) $6x^2$ E) x^2
4. $\lim_{\alpha \rightarrow 1} \frac{4\sqrt{\alpha} - 1}{3\sqrt{\alpha} - 1} = ?$
 A) $-\frac{4}{3}$ B) $-\frac{3}{4}$ C) 1 D) $\frac{3}{4}$ E) $\frac{4}{3}$
5. $\lim_{x \rightarrow 1} (4x - \ln x) = ?$
 A) 1 B) 2 C) 3 D) 4 E) 5
6. $\lim_{x \rightarrow c} \frac{x\sqrt{x} - c\sqrt{c}}{\sqrt{x} - \sqrt{c}} = ?$
 A) $\frac{c}{3}$ B) $\frac{c}{2}$ C) c D) 2c E) 3c
7. $\lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6} - 2}{x - 2} = ?$
 A) $-\frac{1}{1}$ B) $-\frac{1}{8}$ C) 0 D) $\frac{1}{12}$ E) $\frac{1}{4}$
8. $\lim_{x \rightarrow 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x - 2} = ?$
 A) $\frac{1}{2}$ B) $\frac{1}{3 \cdot \sqrt[3]{6}}$ C) $\frac{1}{3 \cdot \sqrt[3]{4}}$
 D) $\frac{-1}{3 \cdot \sqrt[3]{2}}$ E) $\frac{1}{6}$
9. $\lim_{x \rightarrow 2} \frac{x - \sqrt{2x}}{\sqrt{2x+5} - 3} = ?$
 A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$
10. $\lim_{x \rightarrow 1} \frac{1 - \sqrt[p]{x}}{1 - \sqrt[q]{x}} = ?$
 A) $-\frac{p}{q}$ B) $-\frac{q}{p}$ C) $\frac{p}{q}$ D) $\frac{q}{p}$ E) p · q

$$11. \lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 3x + 6} - \sqrt{x^2 - 2x + 4}}{4x - 8} = ?$$

- A) $-\frac{1}{4}$ B) $-\frac{1}{8}$ C) $-\frac{1}{16}$ D) $\frac{1}{8}$ E) $\frac{1}{16}$

$$12. \lim_{x \rightarrow 2} \frac{8 - x^3}{x^2 - 2x + 3} = ?$$

- A) -2 B) -1 C) 0 D) 1 E) 2

$$13. \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{\sqrt[4]{x} - \sqrt{3}} = ?$$

- A) $2\sqrt{3}$ B) $\sqrt{15}$ C) 4 D) $3\sqrt{2}$ E) $3\sqrt{3}$

$$14. \lim_{\frac{a}{b} \rightarrow 1} \frac{4a + 3b}{2a + b} = ?$$

- A) $\frac{2}{7}$ B) $\frac{3}{7}$ C) 2 D) $\frac{7}{3}$ E) $\frac{7}{2}$

$$15. \lim_{x \rightarrow 0} \frac{\sqrt[3]{x + 27} - 3}{\sqrt{x + 64} - 8} = ?$$

- A) $-\frac{27}{22}$ B) $-\frac{32}{27}$ C) $\frac{27}{32}$
D) $\frac{32}{27}$ E) $\frac{16}{27}$

$$16. \lim_{x \rightarrow 5} \left(\frac{1}{x-5} - \frac{3x-8}{x^2-3x-10} \right) = ?$$

- A) $-\frac{5}{7}$ B) $-\frac{2}{7}$ C) 0 D) $\frac{2}{7}$ E) $\frac{5}{7}$

$$17. \lim_{x \rightarrow 1} \frac{ax - \sqrt{x+3}}{x^2 - 1} = b, b \in \mathbb{R} \Rightarrow a = ?$$

- A) 2 B) 3 C) 4 D) 5 E) 6

$$18. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\pi - 2x}{\cos x} = ?$$

- A) -2 B) -1 C) 0 D) 1 E) 2

$$19. \lim_{y \rightarrow x} \frac{\sin^2 y - \sin^2 x}{y^2 - x^2} = ?$$

- A) $\sin x$ B) $\frac{\sin x}{2x}$ C) $\frac{\sin 2x}{x}$
D) $\frac{\sin x}{x}$ E) $\frac{\sin 2x}{2x}$

Yanıtlar / Answers

1. D	2. B	3. B	4. D	5. D	6. E
7. D	8. C	9. C	10. D	11. A	12. C
13. A	14. D	15. E	16. B	17. A	18. E
19. E					

1. $\lim_{x \rightarrow \pi} (\cos(\sin x)) = ?$

- A) 1 B) 0 C) -1 D) $-\frac{1}{2}$ E) $-\frac{1}{3}$

2. $\lim_{x \rightarrow -1} \frac{(2+3x)^2 - (x+2)^2}{x^3 - x} = ?$

- A) -4 B) -2 C) 0 D) 2 E) 4

3. $\lim_{x \rightarrow -1} \frac{3x^3 + 3x^2 + x + 1}{x^2 - 1} = ?$

- A) -3 B) -2 C) -1 D) 0 E) 1

4. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{x+6} - 3} = ?$

- A) 4 B) 9 C) 16 D) 36 E) 39

5. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin 2x - \cos x}{x - \frac{\pi}{2}} = ?$

- A) -1 B) -2 C) -3 D) 1 E) 2

6. $\lim_{x \rightarrow 1} \left(\frac{x+m}{\sqrt{3+x-2}} \right) = k, k \in \mathbb{R} \Rightarrow m = ?$

- A) -1 B) -2 C) -3 D) -4 E) -5

7. $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1 + \cos 2x}{1 - \sin x} \right) = ?$

- A) 0 B) 2 C) -2 D) 4 E) -4

8. $\lim_{x \rightarrow 0} \left(\frac{x^2 + \sin^2 x}{\tan^2 x} \right) = ?$

- A) 3 B) $\frac{3}{2}$ C) 2 D) 1 E) $\frac{5}{2}$

9. $\lim_{x \rightarrow 2} \left(\frac{x^3 - ax^2 + 3x - 2}{x^2 - 4} \right) = k, k \in \mathbb{R} \Rightarrow k = ?$

- A) 3 B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) 2 E) $-\frac{3}{2}$

10. $\lim_{x \rightarrow 1} \left(\frac{\sqrt{x+3} - \sqrt{3x+1}}{x^2 - 3x + 2} \right) = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 0 E) -1

11. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{1 - \tan^2 x} = ?$

- A) -2 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{4}$ E) $\frac{1}{2}$

12. $\lim_{x \rightarrow 0} \frac{5e^x + 5e^{-x} - 10}{4x^2} = ?$

- A) $-\frac{5}{2}$ B) -1 C) 0 D) $\frac{5}{4}$ E) 1

13. $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{6}{x^2-9} \right) = ?$

- A) $\frac{1}{4}$ B) 4 C) 3 D) $\frac{1}{3}$ E) $\frac{1}{6}$

14. $\lim_{x \rightarrow \infty} \frac{(b-1)x^3 + (2a-6)x^2 + x - 1}{ax^2 - 2x + 5} = -1$

$\Rightarrow a + b = ?$

- A) 3 B) 2 C) 1 D) 0 E) ∞

15. $\lim_{x \rightarrow \infty} \left(\frac{\sin x}{x} + \frac{x^2 + 3x}{2x^2 - 1} \right) = ?$

- A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) 1 D) 2 E) ∞

16. $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 1} - \sqrt{x^2 + 3x + 1}) = ?$

- A) -2 B) $-\frac{5}{2}$ C) $-\frac{3}{2}$ D) 0 E) -1

17. $\lim_{x \rightarrow \infty} \left(\frac{x}{x+2} \right)^x = ?$

- A) e^{-2} B) e^2 C) e D) 0 E) -1

18. $\lim_{x \rightarrow \infty} \left(\frac{x-2}{x+2} \right)^{x+2} = ?$

- A) e^4 B) e^{-4} C) e^2 D) e^{-2} E) e

19. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin \left(x - \frac{\pi}{2} \right)}{\cos x} = k, \quad k \in \mathbb{R}$

$\Rightarrow \lim_{x \rightarrow k} (3^{x+1} + e^{-x}) = ?$

- A) e B) $e+3$ C) $e+1$ D) $e+2$ E) $2e$

20. $\lim_{x \rightarrow 0} \frac{\ln(3x+1)}{2x} = ?$

- A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) 3 E) 12

Yanıtlar / Answers

1. C	2. A	3. B	4. D	5. A	6. A
7. D	8. C	9. B	10. B	11. D	12. D
13. E	14. A	15. B	16. C	17. A	18. B
19. C	20. A				

1. $\lim_{x \rightarrow 3} [4 \cdot (3x - 2) \cdot (x + 2)] = ?$

- A) 100 B) 120 C) 135 D) 140 E) 150

2. $\lim_{x \rightarrow -5} [(x + 4)^{100} \cdot (x + 2)] = ?$

- A) -3 B) -2 C) -1 D) 2 E) 3

3. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{(x^3 - 1)^2} = ?$

- A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) 1 D) 3 E) 9

4. $\lim_{a \rightarrow 0} \frac{(x + a)^3 - x^3}{a} = ?$

- A) $\frac{x}{3}$ B) $3x$ C) $3x^2$ D) $6x$ E) $9x^2$

5. $\lim_{x \rightarrow 2} \left(\frac{1}{(x-2)} - \frac{1}{x^2 - 3x + 2} \right) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

6. $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{\sqrt[4]{x} - 2} = ?$

- A) -16 B) -4 C) 0 D) 4 E) 16

7. $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - x}{3 - \sqrt{4x+1}} = ?$

- A) 1 B) $\frac{9}{8}$ C) $\frac{5}{4}$ D) $\frac{11}{8}$ E) $\frac{4}{3}$

8. $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + m} - 1}{x} = n, m, n \in \mathbb{R} \Rightarrow n = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

9. $\lim_{x \rightarrow \infty} \left(\frac{x^2}{2x + 5} - \frac{x}{2} \right) = ?$

- A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) $-\frac{3}{4}$ D) -1 E) $-\frac{5}{4}$

10. $\lim_{x \rightarrow \infty} \left(\frac{x^3}{x^2 + 2} - x \right) = ?$

- A) 0 B) 1 C) 2 D) 3 E) 4

$$11. \lim_{x \rightarrow -\infty} \frac{4}{\frac{x}{1-3^{1-x}}} = ?$$

- A) 4 B) 6 C) 8 D) 10 E) 12

$$12. \lim_{x \rightarrow -\infty} \frac{3 \cdot 2^x + 7}{4 - 2^{x+1}} = ?$$

- A) $\frac{3}{2}$ B) $\frac{13}{5}$ C) $\frac{7}{4}$ D) $\frac{21}{8}$ E) 5

$$13. \lim_{x \rightarrow \infty} (\sqrt{x^2 - 8x - x}) = ?$$

- A) -8 B) -4 C) -2 D) 4 E) 8

$$14. \lim_{x \rightarrow m} \frac{\sin m - \sin x}{\cos x - \cos m} = ?$$

- A) $\cot m$ B) $-\cos m$ C) $-\frac{1}{\cos m}$
D) $-\frac{1}{\sin m}$ E) $-\sin m$

$$15. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin(x - \frac{\pi}{4})}{\cos(x + \frac{\pi}{4})} = ?$$

- A) $-\frac{1}{3}$ B) -1 C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{2}}{2}$

$$16. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cot x - 1} = ?$$

- A) $-2\sqrt{2}$ B) $-\sqrt{2}$ C) $-\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{2}}{2}$ E) $\sqrt{2}$

$$17. \lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2\cos x}{\sin(x - \frac{\pi}{3})} = ?$$

- A) $\frac{\sqrt{2}}{2}$ B) 1 C) $\sqrt{2}$ D) $\frac{3}{2}$ E) $\sqrt{3}$

$$18. \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x^2} - \sqrt[4]{1-2x}}{x} = ?$$

- A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 1 E) 2

$$19. \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + x + 2}{x^4 - 4x - 8} = ?$$

- A) $\frac{1}{64}$ B) $\frac{1}{32}$ C) $\frac{1}{28}$ D) $\frac{1}{12}$ E) $\frac{1}{6}$

Yanıtlar / Answers

1. D	2. A	3. A	4. C	5. D	6. D
7. B	8. C	9. E	10. A	11. B	12. C
13. B	14. A	15. B	16. C	17. E	18. B
19. C					

1. $\lim_{x \rightarrow 0} \frac{\sin^3 \frac{x}{2}}{x^3} = ?$

- A) 16 B) 8 C) 4 D) $\frac{1}{8}$ E) $\frac{1}{16}$

2. $\lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{\sqrt{x} - 2\sqrt{2}} = ?$

- A) $\frac{\sqrt{2}}{3}$ B) $\frac{\sqrt{2}}{6}$ C) $\frac{\sqrt{2}}{8}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

3. $\lim_{x \rightarrow \pi} \frac{\pi \cos 2x - x}{1 + \cos x} = ?$

- A) -1 B) -2 C) 0 D) 1 E) 2

4. $\lim_{x \rightarrow 2} \frac{x - 2}{x^3 - 8} = ?$

- A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) $\frac{1}{8}$ D) $\frac{1}{12}$ E) $\frac{3}{16}$

5. $\lim_{a \rightarrow x} \frac{a^3 - x^3}{a^2 - x^2} = ?$

- A) $\frac{3x^2}{2}$ B) $\frac{3x}{2}$ C) $\frac{3}{2x}$ D) $\frac{x}{3}$ E) $\frac{x}{2}$

6. $\lim_{x \rightarrow 1} \frac{\tan \frac{\pi}{4} x - \cos 2\pi}{x - 1} = ?$

- A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{3\pi}{4}$ E) $\frac{\pi}{6}$

7. $\lim_{x \rightarrow 0} \frac{\sin 4x}{x} = ?$

- A) 1 B) 2 C) 4 D) $\frac{1}{4}$ E) $\frac{1}{8}$

8. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x} = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{3}{2}$ E) $\frac{3}{4}$

9. $\lim_{x \rightarrow \infty} \left(\frac{x+7}{x+3} \right)^x = ?$

- A) $\frac{1}{4}$ B) e^2 C) e^3 D) e^4 E) $\frac{1}{4}e$

10. $\lim_{x \rightarrow \infty} \left(\frac{x+5}{x+1} \right)^{3x+2} = ?$

- A) e B) e^3 C) e^4 D) e^6 E) e^{12}

11. $\lim_{x \rightarrow 2} \frac{x-2}{\sqrt{2x}-2} = ?$

- A) 1 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

12. $\lim_{x \rightarrow e} \frac{\ln x - 1}{x - e} = ?$

- A) 1 B) $\frac{1}{2}$ C) e D) $\frac{1}{e}$ E) $\frac{1}{e^2}$

13. $\lim_{x \rightarrow 0} \frac{\ln(1+4x)}{\sin 4x} = ?$

- A) 16 B) 8 C) 4 D) 2 E) 1

14. $\lim_{x \rightarrow 1} \frac{\sin \pi x}{1-x^2} = ?$

- A) $-\pi$ B) $-\frac{\pi}{2}$ C) $\frac{\pi}{2}$ D) π E) 2π

15. $\lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{x^2} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

16. $\lim_{x \rightarrow \frac{\pi}{3}} \frac{3x-\pi}{\sin\left(x-\frac{\pi}{3}\right)} = ?$

- A) -9 B) -3 C) 0 D) 3 E) 9

17. $\lim_{x \rightarrow \pi} \frac{2x-2\pi}{\tan x - \sin x} = ?$

- A) 2 B) 1 C) 0 D) -1 E) -2

18. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\cos x)}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} = ?$

- A) $-\sqrt{2}$ B) $\frac{-\sqrt{2}}{2}$ C) 1
D) $\frac{\sqrt{2}}{2}$ E) $\sqrt{2}$

19. $\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 2x}{\sin x} = ?$

- A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

20. $\lim_{x \rightarrow \infty} \left(\frac{4x^2+6}{2x^2-7x+1} + 5^{\frac{1}{x}} \right) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 3

Yanıtlar / Answers					
1. D	2. A	3. A	4. D	5. B	6. B
7. C	8. B	9. D	10. E	11. B	12. D
13. E	14. C	15. D	16. D	17. B	18. C
19. E	20. E				

1. $\lim_{t \rightarrow \infty} \left(\frac{t^3 - 2}{t^3 + 1} \right)^{t^3 + 3} = ?$

- A) 1 B) 2 C) e D) e^{-2} E) ∞

2. $\lim_{x \rightarrow -\infty} \left(\sqrt{x^2 - 4x + 6} - \sqrt{x^2 + ax + 3} \right) = 6 \Rightarrow a = ?$

- A) -16 B) -12 C) -8 D) 8 E) 16

3. $\lim_{x \rightarrow 6} \frac{\sqrt{5x+6} - 6}{\sqrt{x+3} - 3} = ?$

- A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) 4 E) $\frac{9}{2}$

4. $\lim_{x \rightarrow 2} (2-x) \cdot \tan\left(\frac{\pi}{4}x\right) = ?$

- A) $\frac{\pi}{2}$ B) $\frac{2}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{2}{\pi}$ E) $\frac{4}{\pi}$

5. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+1} - 1}{\sqrt{x+1} - 1} = ?$

- A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{2}{3}$ E) $\frac{3}{4}$

6. $\lim_{x \rightarrow 0} \frac{\tan(x^2) + \tan^2 x}{x^2} = ?$

- A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

7. $\lim_{x \rightarrow -4} \left(\frac{1}{x+4} - \frac{8}{16-x^2} \right) = ?$

- A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $-\frac{1}{4}$ E) $-\frac{1}{8}$

8. $\lim_{x \rightarrow \frac{\pi}{4}} \tan(2x)(\tan x - 1) = ?$

- A) -1 B) $-\frac{1}{2}$ C) 0 D) 1 E) $\frac{1}{2}$

9. $\lim_{x \rightarrow 0} \frac{\sqrt{x+16} - 4}{\sin 8x} = ?$

- A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{1}{16}$ D) $\frac{1}{32}$ E) $\frac{1}{64}$

10. $\lim_{x \rightarrow 0} \cot x (\operatorname{cosec} x - \cot x) = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{8}$ E) $\frac{3}{4}$

11. $\lim_{x \rightarrow 4} \frac{x^2 - 16}{\sqrt{x-1} - \sqrt{3}} = ?$

- A) $2\sqrt{3}$ B) $4\sqrt{3}$ C) $8\sqrt{3}$ D) $12\sqrt{3}$ E) $16\sqrt{3}$

12. $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25} = ?$

- A) $\frac{1}{5}$ B) $\frac{1}{10}$ C) $\frac{1}{15}$ D) $\frac{1}{20}$ E) $\frac{1}{25}$

13. $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{12}{x^3 - 8} \right) = ?$

- A) $\frac{3}{2}$ B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

14. $\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 2x} - x \right) = ?$

- A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 4

15. $\lim_{x \rightarrow 2} \frac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}} = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

16. $\lim_{x \rightarrow 2} \sqrt{\frac{x^4 - 16}{x^3 - 8}} = ?$

- A) $\frac{\sqrt{6}}{3}$ B) $\frac{2\sqrt{6}}{3}$ C) $\frac{2\sqrt{3}}{3}$
D) $\frac{\sqrt{6}}{4}$ E) $2\sqrt{2}$

17. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\sin^2 x - 1} = ?$

- A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 3

18. $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^2 - 16} = ?$

- A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{1}{16}$ D) $\frac{1}{32}$ E) $\frac{1}{64}$

19. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{4 - 4\sin x}{\sin 4x} = ?$

- A) 2 B) 1 C) 0 D) -1 E) -2

20. $\lim_{x \rightarrow 0} \frac{8x + \sin(6x)}{x^2 - 4x + \sin(12x)} = ?$

- A) $\frac{3}{2}$ B) $\frac{7}{2}$ C) $\frac{7}{4}$ D) $\frac{1}{2}$ E) 1

Yanıtlar / Answers					
1. E	2. D	3. A	4. E	5. D	6. C
7. E	8. A	9. E	10. B	11. E	12. B
13. C	14. A	15. E	16. B	17. B	18. D
19. C	20. C				

1. $\lim_{x \rightarrow -2} (4 - x^2) \cdot \tan\left(\frac{\pi x}{4}\right) = ?$

- A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{8}$ E) $\frac{\pi}{12}$

2. $\lim_{x \rightarrow 3} \frac{\tan(27 - x^3)}{x^4 - 81} = ?$

- A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{4}$ E) $\frac{3}{8}$

3. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{(1 + \sin x) \cdot \cos x}{\pi + 2x} = ?$

- A) 1 B) $\frac{1}{2}$ C) 0 D) $-\frac{1}{2}$ E) -1

4. $\lim_{x \rightarrow 2} \frac{\sin\left(\frac{\pi \cdot x}{4} - \frac{\pi}{2}\right)}{\ln(3x - 5)} = ?$

- A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{3\pi}{4}$ E) $\frac{\pi}{12}$

5. $\lim_{x \rightarrow 0} \frac{\sin(e^{6x} - 1)}{\sin(e^{2x} - 1)} = ?$

- A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 3 E) 6

6. $\lim_{x \rightarrow 1} \sec\left(\frac{\pi}{2}x\right) \cdot \left(\arctan x - \frac{\pi}{4}\right) = ?$

- A) -1 B) $-\frac{1}{2}$ C) $-\frac{1}{\pi}$ D) $\frac{2}{\pi}$ E) $\frac{\pi}{2}$

7. $\lim_{x \rightarrow 0} \frac{2\sin(5x)}{3x} = ?$

- A) $\frac{12}{5}$ B) 3 C) $\frac{10}{3}$ D) $\frac{7}{2}$ E) 4

8. $\lim_{x \rightarrow 0} \sin(5x) \cdot \cot(3x) = ?$

- A) $\frac{4}{3}$ B) $\frac{3}{2}$ C) $\frac{5}{3}$ D) $\frac{7}{3}$ E) $\frac{5}{2}$

9. $\lim_{x \rightarrow 0} x \cdot \operatorname{cosec}^2 \sqrt{2}x = ?$

- A) $\frac{3}{2}$ B) 1 C) $\frac{3}{4}$ D) $\frac{2}{3}$ E) $\frac{1}{2}$

10. $\lim_{x \rightarrow 0} \frac{\sin 2x}{2x^2 + x} = ?$

- A) $\frac{5}{2}$ B) 2 C) $\frac{3}{2}$ D) $\frac{2}{3}$ E) 0

11. $\lim_{x \rightarrow 0} x \cdot \cot(2x) = ?$

- A) $\frac{3}{2}$ B) 1 C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

12. $\lim_{x \rightarrow 0} \frac{x^2 + 4x}{\sin(3x)} = ?$

- A) $\frac{3}{2}$ B) $\frac{4}{3}$ C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) 0

13. $\lim_{x \rightarrow 0} \tan 3x \cdot \cos 6x = ?$

- A) 0 B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) 3 E) $\frac{9}{2}$

14. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{4x - 2\pi}{\cos x} = ?$

- A) 4 B) 2 C) 0 D) -2 E) -4

15. $f(x) = \frac{\sqrt[3]{x} + 2}{x + 8} \Rightarrow \lim_{x \rightarrow -8} f(x) = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{8}$ E) $\frac{1}{12}$

16. $\lim_{x \rightarrow 0} \frac{\sin 2x \cdot \tan 2x}{1 - \cos 4x} = ?$

- A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 4 E) 2

17. $\lim_{x \rightarrow 1} \frac{x^5 - x}{x^2 + 7x - 8} = ?$

- A) $-\frac{2}{3}$ B) $-\frac{5}{6}$ C) $\frac{4}{3}$ D) $\frac{2}{3}$ E) $\frac{4}{9}$

18. $\lim_{x \rightarrow 8} \frac{x - 8}{\sqrt[3]{x - 16} + 12} = ?$

- A) $\frac{5}{6}$ B) $\frac{4}{3}$ C) 6 D) 12 E) 18

19. $\lim_{x \rightarrow 0} \frac{2x + \sin 4x}{\sin 8x} = ?$

- A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) 1 E) $\frac{2}{3}$

20. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin^3 x} = ?$

- A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

Yanıtlar / Answers

1. D	2. A	3. E	4. E	5. D	6. C
7. C	8. C	9. E	10. B	11. D	12. B
13. A	14. E	15. E	16. C	17. E	18. D
19. C	20. B				

Tanım:

$$\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \text{ limitine } f(x) \text{ in } x_0 \text{ noktasındaki türevi}$$

denir.

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx}$$

 $f(x)$ in x_0 noktasındaki türevi,

$f'(x_0)$, $\frac{df(x_0)}{dx}$, $\frac{dy(x_0)}{dx}$, $y'(x_0)$ simgelerinden biri ile gösterilir.

Definition:

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(0)}{x - x_0}$$

The expression above is called the derivative of $f(x)$ at the point x_0 .

$$f'(x) = \frac{df(x)}{dx} = \frac{d_y}{d_x}$$

The derivative of $f(x)$ at $x = x_0$ can be denoted by the followings.

$$f'(x_0), \frac{df(x_0)}{dx}, \frac{dy(x_0)}{dx}, y'(x_0)$$

$$\text{Eş } x - x_0 = h$$

$$\Rightarrow x \rightarrow x_0 \Leftrightarrow h \rightarrow 0$$

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$\text{Eş } f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Örnek / Example:

$$f: \mathbb{R} \rightarrow \mathbb{R},$$

$$f'(x) = x^3 - 4x^2 + 2x - 3 \Rightarrow f'(2) = ?$$

Çözüm / Solution:

$$f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 2x - 3 + 7}{x - 2}$$

$$f'(2) = \lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 2x + 4}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2) \cdot (x^2 - 2x - 2)}{x - 2}$$

$$f'(2) = \lim_{x \rightarrow 2} (x^2 - 2x - 2) = 4 - 4 - 2 = -2, \quad f'(2) = -2$$

Örnek / Example:

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2 - 4x + 4 \Rightarrow f'(x) = \frac{df(x)}{dx} = ?$$

Çözüm / Solution:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - 4x - 4h + 4 - x^2 + 4x - 4}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x - 4 + h)}{h} = 2x - 4,$$

$$f'(x) = x^2 - 4x + 4 \Rightarrow f'(x) = 2x - 4$$



TÜREV ALMA KURALLARI

RULES for TAKING DERIVATIVE

1. $f(x) = c \Rightarrow f'(x) = 0 \ (c \in \mathbb{R})$

2. $f(x) = x^n \Rightarrow f'(x) = n \cdot x^{n-1}$

3. $(f(x) \cdot g(x))' = (f'(x) \cdot g(x)) + (f(x) \cdot g'(x))$

4. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

5. $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

6. $(k \cdot f(x))' = k \cdot f'(x) \ (k \in \mathbb{R})$

7. $(f^m(x))' = m \cdot f^{m-1}(x) \cdot f'(x)$

8. $\left(\sqrt[n]{f(x)}\right)' = \frac{f'(x)}{n \cdot \sqrt[n-1]{f(x)}}$

9. $\left(\sqrt{f(x)}\right)' = \frac{f'(x)}{2 \cdot \sqrt{f(x)}}$

10. $|f(x)|' = \frac{f'(x) \cdot |f(x)|}{f(x)}$

$$|f(x)|' = \begin{cases} f'(x), & f(x) > 0 \\ -f'(x), & f(x) < 0 \end{cases}$$

11. $(f(u(x)))' = u'(x) \cdot f'(u(x))$

Örnekler / Examples:

1. $f(x) = 5 \Rightarrow f'(x) = 0$

2. $f(x) = x \Rightarrow f'(x) = 1$

3. $f(x) = 7x \Rightarrow f'(x) = 7$

4. $f(x) = x^7 \Rightarrow f'(x) = 7x^6$

5. $f(x) = -3x^5 \Rightarrow f'(x) = -15x^4$

6. $f(x) = 2x^3 - 5x^2 + 6x - 7 \Rightarrow f'(x) = 6x^2 - 10x + 6$

7. $f(x) = (x^2 + 2) \cdot (x^3 + x + 1)$

$$f'(x) = 2x \cdot (x^3 + x + 1) + (x^2 + 2) \cdot (3x^2 + 1)$$

8. $f(x) = \frac{x}{x^2 + 3} \Rightarrow f'(x) = \frac{1(x^2 + 3) - 2x(x)}{(x^2 + 3)^2}$

$$= \frac{x^2 + 3 - 2x^2}{(x^2 + 3)^2} = \frac{3 - x^2}{(x^2 + 3)^2}$$

9. $f(x) = \sqrt[5]{x^2 + 2x + 3} \Rightarrow f'(x) = (x^2 + 2x + 3)^{-\frac{1}{5}}$

$$\Rightarrow f'(x) = (2x + 2) \cdot \frac{1}{5} (x^2 + 2x + 3)^{-\frac{4}{5}}$$

$$\Rightarrow f'(x) = \frac{2(x + 1)}{5 \cdot \sqrt[5]{(x^2 + 2x + 3)^4}}$$

10. $f(x) = |x^2 - 5x + 6| \Rightarrow f'(2)$ ve $f'(3)$ yok.

Çünkü, $g(x) = x^2 - 5x + 6$ fonksiyonu $x = 2$ ve $x = 3$ için 0 dır.

$f'(2)$ and $f'(3)$ do not exist because $g(2) = 0$ and $g(3) = 0$.

$$f'(1) = ? \Rightarrow f'(x) = 2x - 5 = -3$$



KAPALI FONKSİYONLARIN TÜREVİ THE DERIVATIVE of CLOSED FUNCTIONS

$$F(x, y) = 0 \Rightarrow \frac{dy}{dx} = -\frac{F'_x(x, y)}{F'_y(x, y)}$$

F'_x : x ' e göre türev, $y \in \mathbb{R}$

F'_y : y ' ye göre türev, $x \in \mathbb{R}$

F'_x : derivative of F with respect to x

F'_y : derivative of F with respect to y

Örnek / Example:

$$F(x, y) = x^4 \cdot y^3 + 2x^3 + 4xy = 0 \Rightarrow y = \frac{dy}{dx} = ?$$

Çözüm / Solution:

$$4x^3 \cdot y^3 + 3y^2 \cdot x^4 \cdot y' + 6x^2 + 4y + 4x \cdot y' = 0$$

$$y'(3x^4 \cdot y^2 + 4x) = -(4x^3 \cdot y^3 + 6x^2 + 4y)$$

$$y' = -\frac{4x^3 \cdot y^3 + 6x^2 + 4y}{3x^4 \cdot y^2 + 4x}$$



BİLEŞKE FONKSİYONUNUN TÜREVİ THE DERIVATIVE of a COMPOSITE FUNCTION

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x)$$

Örnek / Example:

$$f(x) = 4x^2 + 2$$

$$g(x) = x^3 - 3$$

$$\Rightarrow (g \circ f)'(x) = ?$$

Çözüm / Solution:

$$(g \circ f)(x) = (4x^2 + 2)^3 - 3$$

$$(g \circ f)'(x) = 3(4x^2 + 2)^2 \cdot 8x$$

$$= 12(2x + 1)^2 \cdot 8x$$

$$= 96x \cdot (2x + 1)^2$$



PARAMETRİK FONKSİYONLARIN TÜREVİ THE DERIVATIVE of PARAMETRIC FUNCTIONS

$$\left. \begin{array}{l} x = f(t) \\ y = g(t) \end{array} \right\} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Örnek / Example:

$$\left. \begin{array}{l} x = 6t - 3t^2 \\ y = 4t^3 + 3t^2 \end{array} \right\} \Rightarrow \frac{dy}{dx} \Big|_{x=2} = ?$$

Çözüm / Solution:

$$f'(x) = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$f'(x) = (12t^2 + 6t) \cdot \frac{1}{6 - 6t}$$

$$f'(x) = 6(2t^2 + t) \cdot \frac{1}{6(1 - t)}$$

$$t = 2 \Rightarrow$$

$$x = 12 - 12 = 0$$

$$f'(0) = (8 + 2) \cdot \frac{1}{6(-1)} = -\frac{5}{3}$$

$$\left. \begin{array}{l} x = f(t) \\ y = g(t) \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \cdot \frac{dt}{dx} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

Örnek / Example:

$$\left. \begin{array}{l} x = 3t^2 + 3t \\ y = t^3 - 3t \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} \Big|_{t=1} = ?$$

Çözüm / Solution:

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{3t^2 - 3}{6t + 3} = \frac{t^2 - 1}{2t + 1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{2t(2t+1) - 2(t^2-1)}{(2t+1)^2} \cdot \frac{1}{6t+3}$$

$$t = 1 \Rightarrow \frac{d^2y}{dx^2} = \frac{2 \cdot 3 - 2 \cdot 0}{(3)^2} \cdot \frac{1}{9}$$

$$\frac{d^2y}{dx^2} = \frac{6}{81} = \frac{2}{27}$$



TRİGONOMETRİK FONKSİYONLARIN TÜREVİ THE DERIVATIVE of TRIGONOMETRIC FUNCTIONS

$$f(x) = \sin x \Rightarrow f'(x) = \cos x$$

$$f(x) = \sin(u(x)) \Rightarrow f'(x) = u'(x) \cdot \cos(u(x))$$

$$f(x) = \sin^n(u(x)) \Rightarrow f'(x) = n \cdot u'(x) \cdot \sin^{n-1}(u(x)) \cdot \cos(u(x))$$

$$f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

$$f(x) = \cos(u(x)) \Rightarrow f'(x) = -u'(x) \cdot \sin(u(x))$$

$$f(x) = \cos^n(u(x)) \Rightarrow f'(x) = -n(u'(x) \cdot \cos^{n-1}(u(x)) \cdot \sin(u(x)))$$

Örnekler / Examples:

$$1. f(x) = \sin^3(x^2 + x)$$

$$\Rightarrow f'(x) = 3(2x+1) \cdot \sin^2(x^2+x) \cdot \cos(x^2+x)$$

$$2. f(x) = \cos^4(3x) \Rightarrow f'(x) = -4 \cdot 3 \cos^3(3x) \cdot \sin(3x)$$

$$\Rightarrow f'(x) = -12 \cos^3(3x) \cdot \sin(3x)$$

$$f(x) = \tan x \Rightarrow f'(x) = (1 + \tan^2 x) = \sec^2 x$$

$$f(x) = \tan(u(x)) \Rightarrow f'(x) = u'(x) (1 + \tan^2(u(x)))$$

$$= u'(x) \cdot \sec^2(u(x))$$

$$f(x) = \tan^n(u(x))$$

$$f'(x) = n \cdot u'(x) \cdot \tan^{n-1}(u(x)) (1 + \tan^2(u(x)))$$

$$= n \cdot u'(x) \tan^{n-1}(u(x)) \cdot \sec^2(u(x))$$

Örnekler / Examples:

$$1. f(x) = \tan 6x \Rightarrow f'(x) = 6(1 + \tan^2 6x) = 6 \sec^2 6x$$

$$2. f(x) = \tan^3(x^2 - 1)$$

$$\Rightarrow f'(x) = 3 \cdot 2x \tan^2(x^2 - 1) \cdot (1 + \tan^2(x^2 - 1))$$

$$\Rightarrow f'(x) = 6x \tan^2(x^2 - 1) \cdot \sec^2(x^2 - 1)$$

$$f(x) = \cot x \Rightarrow f'(x) = -(1 + \cot^2 x) = -\operatorname{cosec}^2 x$$

$$f(x) = \cot(u(x)) \Rightarrow f'(x) = -u'(x)(1 + \cot^2(u(x)))$$

$$f'(x) = -u'(x) \cdot \operatorname{cosec}^2(u(x))$$

$$f(x) = \cot^n(u(x)) \Rightarrow f'(x) = -n(u'(x)) \cot^{n-1}(u(x)) \cdot (1 + \cot^2(u(x)))$$

$$\Rightarrow f'(x) = -n \cdot (u'(x)) \cot^{n-1}(u(x)) \cdot \operatorname{cosec}^2(u(x))$$

Örnekler / Examples:

$$1. f(x) = \cot 4x \Rightarrow f'(x) = -4(1 + \cot^2 4x) = -4 \operatorname{cosec}^2 4x$$

$$2. f(x) = \cot^2(x^2 + 3x)$$

$$\Rightarrow f'(x) = -2(2x+3) \cdot \cot(x^2+3x) (1 + \cot^2(x^2+3x))$$

$$\Rightarrow f'(x) = -2(2x+3) \cdot \cot(x^2+3x) \cdot \operatorname{cosec}^2(x^2+3x)$$



TERS TRİGONOMETRİK FONKSİYONLARIN TÜREVİ

THE DERIVATIVE of INVERSE TRIGONOMETRIC FUNCTIONS

$$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$f(x) = \arcsin(u(x)) \Rightarrow f'(x) = \frac{u'(x)}{\sqrt{1-(u(x))^2}}$$

$$f(x) = \arccos x \Rightarrow f'(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$f(x) = \arccos(u(x)) \Rightarrow f'(x) = \frac{-u'(x)}{\sqrt{1-(u(x))^2}}$$

$$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$$

$$f(x) = \arctan(u(x)) \Rightarrow f'(x) = \frac{u'(x)}{1+(u(x))^2}$$

$$f(x) = \operatorname{arccot} x \Rightarrow f'(x) = \frac{-1}{1+x^2}$$

$$f(x) = \operatorname{arccot}(u(x)) \Rightarrow f'(x) = \frac{-u'(x)}{1+(u(x))^2}$$

Örnekler / Examples:

$$1. f(x) = \arctan(x^2 + 2x) - \arccos(2x) \Rightarrow f'(0) = ?$$

$$f'(x) = \frac{2x+2}{1+(x^2+2x)^2} - \frac{-2x}{\sqrt{1-(2x)^2}}$$

$$f'(x) = \frac{2(x+1)}{1+(x^2+2x)^2} + \frac{2x}{\sqrt{1-4x^2}}$$

$$f'(0) = \frac{2(0+1)}{1+0^2} + \frac{2 \cdot 0}{\sqrt{1-0}} \Rightarrow f'(0) = 2$$

$$2. f(x) = \operatorname{arccot}(x^2 + 2x) - (\arcsin x)^2 \Rightarrow f'\left(\frac{1}{2}\right) = ?$$

$$f'(x) = \frac{-(2x+2)}{1+(x^2+2x)^2} - \frac{1}{\sqrt{1-x^2}} \cdot 2 \arcsin x$$

$$f'\left(\frac{1}{2}\right) = \frac{-\left(2 \cdot \frac{1}{2} + 2\right)}{1+\left(\frac{1}{4} + 2 \cdot \frac{1}{2}\right)^2} - \frac{1}{\sqrt{1-\frac{1}{4}}} \cdot 2 \arcsin \frac{1}{2}$$

$$f'\left(\frac{1}{2}\right) = \frac{-3}{1+\frac{25}{16}} - \frac{2}{\sqrt{3}} \cdot 2 \cdot \frac{\pi}{6}$$

$$= \frac{-48}{41} - \frac{2\sqrt{3}\pi}{9} = -\frac{432 + 82\sqrt{3}\pi}{369}$$



LOGARİTMİK FONKSİYONLARIN TÜREVİ THE DERIVATIVE of LOGARITHMIC FUNCTIONS

$$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \cdot \ln a} = \frac{1}{x} \cdot \log_a e$$

$$f(x) = \log_a (u(x)) \Rightarrow f'(x) = \frac{u'(x)}{u(x) \cdot \ln a} = \frac{u'(x)}{u(x)} \cdot \log_a e$$

$$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$$

$$f(x) = \ln(u(x)) \Rightarrow f'(x) = \frac{u'(x)}{u(x)}$$

Örnekler / Examples:

1. $f(x) = \log_5 (x^2 + 4x) \Rightarrow f'(2) = ?$

$$f'(x) = \frac{2x + 4}{x^2 + 4x} \log_5 e \Rightarrow f'(2) = \frac{8}{12} \cdot \log_5 e$$

$$\Rightarrow f'(2) = \frac{2}{3} \cdot \log_5 e$$

2. $f(x) = \ln(x^3 + 6x) \Rightarrow f'(1) = ?$

$$f'(x) = \frac{3x^2 + 6}{x^3 + 6x} \Rightarrow f'(1) = \frac{3 \cdot 1 + 6}{1 + 6} = \frac{9}{7}$$



ÜSTEL FONKSİYONLARIN TÜREVİ THE DERIVATIVE of EXPONENTIAL FUNCTIONS

$$f(x) = e^x \Rightarrow f'(x) = e^x$$

$$f(x) = e^{u(x)} \Rightarrow f'(x) = u'(x) \cdot e^{u(x)}$$

$$f(x) = a^x \Rightarrow f'(x) = a^x \cdot \ln a = \frac{1}{\log_a e} \cdot a^x$$

$$f(x) = a^{u(x)} \Rightarrow f'(x) = u'(x) \cdot a^{u(x)} \cdot \ln a = \frac{u'(x)}{\log_a e} a^{u(x)}$$

Örnekler / Examples:

1. $f(x) = e^{\ln x} \Rightarrow f'(e) = ?$

$$f(x) = e^{\ln x} \Rightarrow f'(x) = \frac{1}{x} \cdot e^{\ln x}$$

$$f'(e) = \frac{1}{e} \cdot e^{\ln e} = \frac{1}{e} \cdot e = 1$$

2. $f(x) = 5^{x^2 + 4} \Rightarrow f'(x) = ?$

$$f'(x) = 2x \cdot 5^{x^2 + 4} \cdot \ln 5$$



YÜKSEK BASAMAKTAN TÜREVLER HIGHER ORDER DERIVATIVES

$$y' = f'(x) \quad 1. \text{ basamaktan türev / 1st order derivative}$$

$$y'' = f''(x) \quad 2. \text{ basamaktan türev / 2nd order derivative}$$

$$y''' = f'''(x) \quad 3. \text{ basamaktan türev / 3rd order derivative}$$

⋮

$$y^{(n)} = f^{(n)}(x) : n. \text{ basamaktan türev / } n^{\text{th}} \text{ order derivative}$$

Örnek / Example:

$$f(x) = x^3 + 4x^2 - 2x + 6 \Rightarrow f''(x) = ?$$

Çözüm / Solution:

$$f'(x) = 3x^2 + 8x - 2 \Rightarrow f''(x) = 6x + 8 \Rightarrow f'''(x) = 6$$

Örnek / Example:

$$f(x, y) = x^2 + y^2 - 9 = 0 \Rightarrow f''(x, y) = ?$$

Çözüm / Solution:

$$y' = f'(x) = \frac{-2x}{2y} = \frac{-x}{y}$$

$$y'' = f''(x) = \frac{-1 \cdot y - xy'}{y^2}$$

$$= \frac{-y + x \left(\frac{-x}{y} \right)}{y^2} = \frac{-y^2 - x^2}{y^3}$$

$$= -\frac{x^2 + y^2}{y^3}$$



L HOSPITAL KURALI / L HOSPITAL RULE

$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)}$ limiti bulunurken

$\frac{0}{0}, \frac{\infty}{\infty}$ şeklinde belirsizlik oluşuyorsa limit, pay ve paydanın ayrı ayrı türevleri alınarak bulunabilir.

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x_0)}{g'(x_0)} \text{ dir.}$$

$\lim_{x \rightarrow x_0} \frac{f'(x_0)}{g'(x_0)}$ limiti tekrar belirsizlik oluşturursa kural

yeniden uygulanır.

If $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)}$ is equal to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then the limit is found by taking the derivatives of the numerator and denominator separately.

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x_0)}{g'(x_0)}$$

If the limit $\lim_{x \rightarrow x_0} \frac{f'(x_0)}{g'(x_0)}$ again equals the same

uncertainty, we apply the rule again, that is, we take the second derivative.

Örnekler / Examples:

$$1. \lim_{x \rightarrow 8} \frac{\sqrt{x} - 2\sqrt{2}}{\sqrt[3]{x} - 2} = \lim_{x \rightarrow 8} \frac{\sqrt{8} - 2\sqrt{2}}{\sqrt[3]{8} - 2} = \frac{2\sqrt{2} - 2\sqrt{2}}{2 - 2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 8} \frac{(\sqrt{x} - 2\sqrt{2})'}{(\sqrt[3]{x} - 2)'} = \lim_{x \rightarrow 8} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{3\sqrt[3]{x^2}}} = \frac{1}{2\sqrt{8}} \cdot \frac{3\sqrt[3]{8}}{1} = \frac{3\sqrt{2}}{4}$$

$$2. \lim_{x \rightarrow e} \frac{e^{\ln x} - e}{\ln(\ln x)} = \lim_{x \rightarrow e} \frac{e^{\ln x} - e}{\ln(\ln x)} = \frac{e - e}{\ln(\ln e)} = \frac{0}{\ln 1} = \frac{0}{0}$$

$$\lim_{x \rightarrow e} \frac{e^{\ln x} - e}{\ln(\ln x)} = \lim_{x \rightarrow e} \frac{\frac{1}{x} e^{\ln x}}{\frac{1}{x \ln x}} = \frac{1}{\frac{1}{e} \cdot 1} = e$$

ÇÖZÜMLÜ TEST / TEST WITH SOLUTIONS

$$1. f(x) = 2x^3 - 5x^2 + 4x - 7 \Rightarrow f'(x) = ?$$

- A) $3x^2 - 5x - 4$
B) $6x^2 - 10x + 4$
C) $6x^2 - 10x - 7$
D) $3x^2 - 5x + 4$
E) $x^3 - x^2 + 4$

Çözüm / Solution:

$$f(x) = 2x^3 - 5x^2 + 4x - 7$$

$$f'(x) = 6x^2 - 10x + 4$$

Yanıt / Answer B

$$2. f(x) = (x^2 - 1) \cdot (2x^2 - 3x + 1) \Rightarrow f'(2) = ?$$

- A) 21 B) 24 C) 27 D) 30 E) 33

Çözüm / Solution:

$$\begin{aligned} f'(x) &= (x^2 - 1)' \cdot (2x^2 - 3x + 1) + (2x^2 - 3x + 1)' \cdot (x^2 - 1) \\ &= 2x \cdot (2x^2 - 3x + 1) + (4x - 3) \cdot (x^2 - 1) \\ f'(2) &= 2 \cdot 2 \cdot (2 \cdot 2^2 - 3 \cdot 2 + 1) + (4 \cdot 2 - 3) \cdot (2^2 - 1) \\ &= 4 \cdot (8 - 6 + 1) + (8 - 3) \cdot (4 - 1) \\ &= 4 \cdot 3 + 5 \cdot 3 \\ &= 12 + 15 \\ &= 27 \end{aligned}$$

Yanıt / Answer C

$$3. f(x) = \frac{x^2 - 3}{3x + 1} \Rightarrow f'(-1) = ?$$

- A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

Çözüm / Solution:

$$f'(x) = \frac{(x^2-3)'(3x+1) - (3x+1)' \cdot (x^2-3)}{(3x+1)^2}$$

$$f'(x) = \frac{2x(3x+1) - 3 \cdot (x^2-3)}{(3x+1)^2}$$

$$f'(x) = \frac{6x^2 + 2x - 3x^2 + 9}{(3x+1)^2}$$

$$f'(-1) = \frac{3 \cdot (-1)^2 + 2 \cdot (-1) + 9}{(3 \cdot (-1) + 1)^2}$$

$$= \frac{3-2+9}{(-2)^2}$$

$$= \frac{10}{4}$$

$$= \frac{5}{2}$$

Yanıt / Answer D

4. $f(x) = \frac{x^3-1}{x} \Rightarrow f'(x) = ?$

A) $3x + \frac{1}{x}$ B) $2x + \frac{1}{x^2}$ C) $3x - \frac{1}{x^2}$

D) $4x + \frac{1}{x^2}$ E) $3x + \frac{1}{x^2}$

Çözüm / Solution:

$$f(x) = \frac{x^3-1}{x}$$

$$f(x) = \frac{x^3}{x} - \frac{1}{x}$$

$$f(x) = x^2 - x^{-1}$$

$$f'(x) = 2x + x^{-2}$$

$$f'(x) = 2x + \frac{1}{x^2}$$

Yanıt / Answer B

5. $f(x) = (x^4 - 2x)^5 \Rightarrow f'(1) = ?$

A) 10 B) 12 C) 14 D) 16 E) 18

Çözüm / Solution:

$$f(x) = (x^4 - 2x)^5$$

$$f'(x) = 5 \cdot (x^4 - 2x)^4 \cdot (x^4 - 2x)'$$
$$= 5 \cdot (x^4 - 2x)^4 \cdot (4x^3 - 2)$$

$$f'(1) = 5 \cdot (1^4 - 2 \cdot 1)^4 \cdot (4 \cdot 1^3 - 2)$$

$$= 5 \cdot (-1)^4 \cdot 2$$

$$= 5 \cdot 1 \cdot 2$$

$$= 10$$

Yanıt / Answer A

6. $f(x) = \sqrt[3]{x^2-3} \Rightarrow f'(2) = ?$

A) $\frac{1}{5}$ B) $\frac{4}{3}$ C) $\frac{1}{2}$ D) 1 E) 2

Çözüm / Solution:

$$f(x) = \sqrt[3]{x^2-3} = (x^2-3)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} \cdot (x^2-3)^{-\frac{2}{3}} \cdot (x^2-3)'$$

$$f'(x) = \frac{1}{3} \cdot (x^2-3)^{-\frac{2}{3}} \cdot (2x)$$

$$f'(2) = \frac{1}{3} \cdot (2^2-3)^{-\frac{2}{3}} \cdot 2 \cdot 2$$

$$= \frac{1}{3} \cdot 1 \cdot 2 \cdot 2$$

$$= \frac{4}{3}$$

Yanıt / Answer B

7. $x^2 + 2xy - y^2 = 0 \Rightarrow \frac{dy}{dx} = ?$

A) $\frac{x-y}{y}$ B) $\frac{y-x}{x+y}$ C) $\frac{x+y}{y-x}$

D) $\frac{x+y}{x-y}$ E) $\frac{x-y}{x+y}$

Çözüm / Solution:

$$x^2 + 2xy - y^2 = 0$$

$$\frac{dy}{dx} = -\frac{2x+2y}{2x-2y}$$

$$= -\frac{2 \cdot (x+y)}{2 \cdot (x-y)}$$

$$= \frac{x+y}{y-x}$$

Yanıt / Answer C

8. $f(4) = -3, f'(4) = 2$ ve $g'(-3) = -5$

$$\Rightarrow (g \circ f)'(4) = ?$$

A) -16 B) -14 C) -12 D) -10 E) -8

Çözüm / Solution:

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x)$$

$$(g \circ f)'(4) = g'(f(4)) \cdot f'(4)$$

$$= g'(-3) \cdot 2$$

$$= (-5) \cdot 2$$

$$= -10$$

Yanıt / Answer D

9. $y = 3x^2 - 1, z = 2y^3 + 4 \Rightarrow \frac{dz}{dx} = ?$

A) $36x(3x^2 - 1)^2$ B) $6x \cdot (3x^2 - 1)^2$

C) $36 \cdot (x^2 - 1)^2$ D) $x(3x^2 - 1)^2$

E) $18x(3x^2 - 1)$

Çözüm / Solution:

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= 6y^2 \cdot 6x$$

$$= 36 \cdot y^2 \cdot x$$

$$= 36 \cdot (3x^2 - 1)^2 \cdot x$$

$$= 36x \cdot (3x^2 - 1)^2$$

Yanıt / Answer A

10. $\left. \begin{array}{l} x = 4t - t^2 \\ y = 2t^2 + t \end{array} \right\} \Rightarrow \frac{dy}{dx} = ?$

A) $\frac{2t+4}{4t-2}$ B) $\frac{4t-1}{2t+4}$ C) $\frac{4t+1}{4-2t}$

D) $\frac{2t-4}{4t+1}$ E) $\frac{t^2-1}{4+2t}$

Çözüm / Solution:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t+1}{4-2t}$$

Yanıt / Answer C

11. $f(x) = \sin(3x+2) \Rightarrow f'(x) = ?$

A) $-2 \cos(3x+2)$

B) $2 \cos(3x+2)$

C) $2 \sin(3x+2)$

D) $3 \cdot \cos(3x+2)$

E) $-3 \cdot \cos(3x+2)$

Çözüm / Solution:

$$f(x) = \sin(3x+2)$$

$$f'(x) = \cos(3x+2) \cdot 3$$

$$= 3 \cdot \cos(3x+2)$$

Yanıt / Answer D

12. $f(x) = \cos^2(3x) \Rightarrow f'\left(\frac{\pi}{6}\right) = ?$

A) $-\frac{1}{4}$

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) $\frac{1}{4}$

Çözüm / Solution:

$$f(x) = \cos^2(3x)$$

$$f'(x) = 2 \cos(3x) \cdot ((\cos 3x))'$$

$$= 2 \cdot \cos(3x) \cdot (-\sin(3x) \cdot 3)$$

$$= -6 \cdot \cos 3x \cdot \sin 3x$$

$$= -3 \cdot 2 \cdot \sin(3x) \cdot \cos(3x)$$

$$\sin 2\alpha = 2 \cdot \sin \alpha \cos \alpha$$

$$= -3 \cdot \sin(6x)$$

$$f'\left(\frac{\pi}{6}\right) = -3 \cdot \sin\left(6 \cdot \frac{\pi}{6}\right)$$

$$= -3 \cdot \sin(\pi)$$

$$= -3 \cdot 0$$

$$= 0$$

Yanıt / Answer C

13. $f(x) = \tan 4x + \cot 2x \Rightarrow f'\left(\frac{2\pi}{3}\right) = ?$

- A) $\frac{32}{3}$ B) 11 C) $\frac{35}{3}$ D) 13 E) $\frac{40}{3}$

Çözüm / Solution:

$$\begin{aligned} f(x) &= \tan 4x + \cot 2x \\ f'(x) &= (1 + \tan^2 4x) \cdot 4 - (1 + \cot^2 2x) \cdot 2 \\ f'\left(\frac{2\pi}{3}\right) &= (1 + \tan^2 \frac{8\pi}{3}) \cdot 4 - (1 + \cot^2 2x) \cdot 2 \\ &= (1 + \tan^2 \frac{2\pi}{3}) \cdot 4 - (1 + \cot^2 \frac{4\pi}{3}) \cdot 2 \\ &= (1 + (-\sqrt{3})^2) \cdot 4 - (1 + \left(\frac{1}{\sqrt{3}}\right)^2) \cdot 2 \\ &= (1 + 3) \cdot 4 - (1 + \frac{1}{3}) \cdot 2 \\ &= 16 - \frac{8}{3} \\ &= \frac{40}{3} \end{aligned}$$

Yanıt / Answer E

14. $f(x) = \sin^3 2x \Rightarrow f'\left(\frac{\pi}{4}\right) = ?$

- A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

Çözüm / Solution:

$$\begin{aligned} f(x) &= \sin^3 2x \\ f'(x) &= 3 \cdot \sin^2 2x \cdot \cos 2x \cdot 2 \\ f'\left(\frac{\pi}{4}\right) &= 3 \cdot \sin^2 \frac{\pi}{2} \cdot \cos \frac{\pi}{2} \cdot 2 = 0 \end{aligned}$$

Yanıt / Answer C

15. $f(x) = \arctan(x^2 - 1) \Rightarrow f'(2) = ?$

- A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) 1 E) $\frac{6}{5}$

Çözüm / Solution:

$$\begin{aligned} f'(x) &= \frac{2x}{1 + (x^2 - 1)^2} \\ f'(2) &= \frac{4}{1 + (2^2 - 1)^2} \\ &= \frac{4}{10} = \frac{2}{5} \end{aligned}$$

Yanıt / Answer A

16. $f(x) = \arctan x + (\arccos x)^2 \Rightarrow f'(0) = ?$

- A) $1 + \pi$ B) $1 - \pi$ C) $2 + \pi$
D) $2 - \pi$ E) $3 + \pi$

Çözüm / Solution:

$$\begin{aligned} f(x) &= \arctan x + (\arccos x)^2 \\ f'(x) &= \frac{1}{1+x^2} + 2 \cdot (\arccos x) \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right) \\ f'(0) &= \frac{1}{1+0} + 2 \cdot (\arccos 0) \cdot \left(-\frac{1}{\sqrt{1+0}}\right) \\ f'(0) &= 1 + 2 \cdot \frac{\pi}{2} \cdot (-1) \\ &= 1 - \pi \end{aligned}$$

Yanıt / Answer B

17. $f(x) = \ln(x^2 + 3x) \Rightarrow f'(x) = ?$

- A) $\frac{2x+2}{x^2+2x}$ B) $\frac{2}{x+3x^2}$ C) $\frac{2x+3}{x^2+3x}$
D) $\frac{2x+3}{x+3}$ E) $\frac{2x}{x^2+3}$

Çözüm / Solution:

$$f'(x) = \frac{2x+3}{x^2+3x}$$

Yanıt / Answer C

18. $f(x) = \ln(\cos x) \Rightarrow f'(x) = ?$

- A) $\sin x$ B) $\cot x$ C) $-\cot x$
D) $-\tan x$ E) $\tan x$

Çözüm / Solution:

$$\begin{aligned} f(x) &= \ln(\cos x) \\ f'(x) &= \frac{-\sin x}{\cos x} \\ f'(x) &= -\tan x \end{aligned}$$

Yanıt / Answer D

19. $f(x) = \log_5(x^2 - 2) \Rightarrow f'(5) = ?$

- A) $\frac{13}{5 \ln 5}$ B) $\frac{23}{2 \ln 5}$ C) $\frac{13}{\ln 5}$
D) $\frac{2}{5 \ln 5}$ E) $\frac{10}{23 \ln 5}$

Çözüm / Solution:

$$f(x) = \log_5 (x^2 - 2)$$

$$f'(x) = \frac{2x}{(x^2 - 2) \cdot \ln 5}$$

$$f'(5) = \frac{2 \cdot 5}{(5^2 - 2) \cdot \ln 5}$$

$$= \frac{10}{23 \cdot \ln 5}$$

Yanıt / Answer E

20. $f(x) = \ln (\operatorname{cosec} x + \cot x) \Rightarrow f'(x) = ?$

A) $\frac{\cot x + \sin x}{\sin x - 1}$ B) $\frac{\sin x}{\sin x - 1}$ C) $\frac{\cos x}{\cos x - 1}$

D) $\sec x$

E) $-\operatorname{cosec} x$

Çözüm / Solution:

$$f(x) = \ln(\operatorname{cosec} x + \cot x)$$

$$f(x) = \ln \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} \right)$$

$$f(x) = \ln \left(\frac{1 + \cos x}{\sin x} \right)$$

$$f'(x) = \frac{\left(\frac{1 + \cos x}{\sin x} \right)'}{\frac{1 + \cos x}{\sin x}}$$

$$= \frac{-\sin x \cdot \sin x - \cos x \cdot (1 + \cos x)}{\frac{(1 + \cos x)^2}{\sin x}}$$

$$= \frac{-\sin^2 x - \cos x - \cos^2 x}{\sin^2 x} \cdot \frac{\sin x}{1 + \cos x}$$

$$= \frac{-(\sin^2 x + \cos^2 x) - \cos x}{\sin^2 x} \cdot \frac{\sin x}{1 + \cos x}$$

$$= \frac{(-1 - \cos x) \cdot \sin x}{\sin^2 x \cdot (1 + \cos x)}$$

$$= \frac{-(1 + \cos x) \cdot \sin x}{\sin^2 x \cdot (1 + \cos x)}$$

$$= -\frac{1}{\sin x}$$

$$= -\operatorname{cosec} x$$

Yanıt / Answer E

21. $f(x) = e^{x^2+1} \Rightarrow f'(1) = ?$

A) $\frac{e^2}{2}$ B) $3e$ C) $2e^2$ D) $2e$ E) $3e^2$

Çözüm / Solution:

$$f(x) = e^{x^2+1}$$

$$f'(x) = e^{x^2+1} \cdot 2x$$

$$f'(1) = e^{2+1} \cdot 2$$

$$f'(1) = 2 \cdot e^2$$

Yanıt / Answer C

22. $f(x) = 5^{\cos x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

A) $\ln \frac{1}{5}$ B) $\ln 5$ C) $\ln \frac{2}{5}$

D) $\ln 25$ E) $\ln \frac{3}{5}$

Çözüm / Solution:

$$f(x) = 5^{\cos x}$$

$$f'(x) = 5^{\cos x} \cdot (-\sin x) \cdot \ln 5$$

$$f'\left(\frac{\pi}{2}\right) = 5^{\cos \frac{\pi}{2}} \cdot \left(-\sin \frac{\pi}{2}\right) \cdot \ln 5$$

$$f'\left(\frac{\pi}{2}\right) = 5^0 \cdot (-1) \cdot \ln 5$$

$$= -\ln 5$$

$$= \ln \frac{1}{5}$$

Yanıt / Answer A

23. $f(x) = x^2 + 3x - 5 \Rightarrow f''(x) = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

Çözüm / Solution:

$$f(x) = x^2 + 3x - 5$$

$$f'(x) = 2x + 3$$

$$f''(x) = 2$$

Yanıt / Answer A

YÖS SORULARI / YÖS QUESTIONS

1. $x = 3t^2 + 6t, y = 2t^3 - 6t \Rightarrow \frac{dy}{dx} = ?$

- A) t B) $t-1$ C) $t+1$
D) $\frac{t-1}{t+1}$ E) $\frac{t+1}{t-1}$

(YÖS 1990)

Çözüm / Solution:

$$\left. \begin{array}{l} x = 3t^2 + 6t \\ y = 2t^3 - 6t \end{array} \right\} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{6t^2 - 6}{6t + 6}$$

$$= \frac{6 \cdot (t^2 - 1)}{6 \cdot (t + 1)}$$

$$= \frac{(t-1) \cdot (t+1)}{t+1} = t-1$$

Yanıt / Answer B

2. $x = e^{3t} \cdot \cos t, y = e^{3t} \sin t \Rightarrow \frac{dy}{dx} \left(\frac{\pi}{4} \right) = ?$

- A) -4 B) -2 C) 2 D) 3 E) 4

(YÖS 1991)

Çözüm / Solution:

$$\left. \begin{array}{l} x = e^{3t} \cdot \cos t \\ y = e^{3t} \cdot \sin t \end{array} \right\} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{e^{3t} \cdot 3 \cdot \sin t + \cos t \cdot e^{3t}}{e^{3t} \cdot 3 \cdot \cos t - \sin t \cdot e^{3t}}$$

$$= \frac{e^{3t} \cdot (3 \sin t + \cos t)}{e^{3t} \cdot (3 \cos t - \sin t)}$$

$$\frac{dy}{dx} \left(\frac{\pi}{4} \right) = \frac{3 \cdot \sin \frac{\pi}{4} + \cos \frac{\pi}{4}}{3 \cos \frac{\pi}{4} - \sin \frac{\pi}{4}}$$

$$= \frac{3 \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{3 \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} = \frac{4 \cdot \frac{\sqrt{2}}{2}}{2 \cdot \frac{\sqrt{2}}{2}} = 2$$

Yanıt / Answer C

3. $f(x) = x \cdot \sqrt{x^2 + 2x - 3} \Rightarrow \sqrt{5} f'(2) = ?$

- A) -3 B) -2 C) 8 D) 10 E) 11

(YÖS 1993)

Çözüm / Solution:

$$f(x) = x \cdot \sqrt{x^2 + 2x - 3}$$

$$f'(x) = 1 \cdot \sqrt{x^2 + 2x - 3} + \frac{2x + 2}{2\sqrt{x^2 + 2x - 3}} \cdot x$$

$$f'(x) = \sqrt{x^2 + 2x - 3} + \frac{2(x+1) \cdot x}{2\sqrt{x^2 + 2x - 3}}$$

$$f'(x) = \sqrt{x^2 + 2x - 3} + \frac{(x+1) \cdot x}{\sqrt{x^2 + 2x - 3}}$$

$$f'(2) = \sqrt{2^2 + 2 \cdot 2 - 3} + \frac{(2+1) \cdot 2}{\sqrt{2^2 + 2 \cdot 2 - 3}}$$

$$f'(2) = \sqrt{5} + \frac{6}{\sqrt{5}}$$

$$\sqrt{5} \cdot f'(2) = \sqrt{5} \cdot \left(\sqrt{5} + \frac{6}{\sqrt{5}} \right) = 5 + 6 = 11$$

Yanıt / Answer E

4. $x = t + \frac{1}{t}, y = t^2 - \frac{2}{t}, t = 2 \Rightarrow \frac{dy}{dx} = ?$

- A) 6 B) 4 C) 2 D) -2 E) -4

(YÖS 1993)

Çözüm / Solution:

$$\left. \begin{array}{l} x = t + \frac{1}{t} \\ y = t^2 - \frac{2}{t} \end{array} \right\} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2t + \frac{2}{t^2}}{1 - \frac{1}{t^2}}$$

$$= \frac{2t^3 + 2}{t^2 - 1} = \frac{2t^3 + 2}{t^2 - 1}$$

$$t = 2 \Rightarrow \frac{dy}{dx} = \frac{2 \cdot 2^3 + 2}{2^2 - 1} = \frac{18}{3} = 6$$

Yanıt / Answer A

5. $\frac{1}{\sin 2x} \cdot \frac{d}{dx} (\sin^2 x) = ?$

- A) $\sin x$ B) $\cos x$ C) $\cos 2x$
D) -1 E) 1

(YÖS 1993)

Çözüm / Solution:

$$\begin{aligned} \frac{1}{\sin 2x} \cdot \frac{d}{dx} (\sin^2 x) &= \frac{1}{\sin 2x} \cdot 2 \cdot \sin x \cdot \cos x \\ &= \frac{1}{\sin 2x} \cdot \sin 2x \\ &= 1 \end{aligned}$$

Yanıt / Answer E

6. $x = 2t - \frac{1}{2}, y = t^2 + 2 \Rightarrow \frac{d^2 y}{dx^2} = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{3}{4}$ E) $\frac{7}{5}$

(YÖS 1994)

Çözüm / Solution:

$$\begin{cases} x = 2t - \frac{1}{2} \\ y = t^2 + 2 \end{cases} \Rightarrow \frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{d}{dx} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right)$$

$$= \frac{d}{dx} \left(\frac{2t}{2} \right)$$

$$= \frac{d}{dx} (t) = \frac{1}{2}$$

Yanıt / Answer B

7. $y = e^{2t}, x = \cos e^{2t} \Rightarrow \frac{dy}{dx} = ?$

- A) $\cos 2x$ B) $\sin 2x$ C) $\sin x$
D) $\frac{1}{\sin(\arcsin x)}$ E) $-\frac{1}{\sin(\arccos x)}$

(YÖS 1995)

Çözüm / Solution:

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2e^{2t}}{-2e^{2t} \sin(e^{2t})} \\ &= \frac{-1}{\sin(e^{2t})} = \frac{-1}{\sin(\arccos x)} \end{aligned}$$

Yanıt / Answer E

8. $f(x) = x \cos x$

$$\left. \frac{d^2 f(x)}{dx^2} \right|_{x=\frac{\pi}{2}}$$

- A) -3 B) -2 C) -1 D) 0 E) 1

(YÖS 1997)

Çözüm / Solution:

$$\begin{aligned} \frac{d}{dx} \left(\frac{df(x)}{dx} \right) &= \frac{d}{dx} (\cos x - \sin x \cdot x) \\ &= -\sin x - (\cos x \cdot x + \sin x) \\ &= -\sin x - x \cdot \cos x - \sin x \\ &= -2 \sin x - x \cdot \cos x \\ x = \frac{\pi}{2} &\Rightarrow -2 \sin \frac{\pi}{2} - \frac{\pi}{2} \cdot \cos \frac{\pi}{2} = -2 \cdot 1 - \frac{\pi}{2} \cdot 0 \\ &= -2 \end{aligned}$$

Yanıt / Answer B

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9.

$$f(x) = \frac{x^2 - 5x + 6}{x^2 + 5x + 6} \Rightarrow \frac{df}{dx}(0) = f'(0) = ?$$

- A) $-\frac{5}{3}$ B) $-\frac{2}{3}$ C) $-\frac{1}{3}$ D) -2 E) -1

(YÖS 2001)

Çözüm / Solution:

$$\begin{aligned} \frac{df}{dx} &= f'(x) \\ &= \frac{(2x-5) \cdot (x^2+5x+6) - (x^2-5x+6)(2x+5)}{(x^2+5x+6)^2} \end{aligned}$$

$$f'(0) = \frac{(-5) \cdot 6 - 6 \cdot 5}{6^2} = \frac{-60}{36} = -\frac{5}{3}$$

Yanıt / Answer A

$$10. y = x \cdot e^{-x} + \ln 2 \Rightarrow \frac{dy}{dx} = ?$$

- A) $x \cdot e^{-x}$ B) $e^{-x} - x \cdot e^{-x}$
C) $e^{-x} - x \cdot e^x + \frac{1}{2}$ D) $e^{-x} + \frac{1}{2}$
E) $x \cdot e^{-x} + 2$

(YÖS 2001)

Çözüm / Solution:

$$\frac{dy}{dx} = e^{-x} - x e^{-x}$$

Yanıt / Answer B

$$11. f(x) = \sqrt{x}(x^3 - 3) \Rightarrow \frac{df}{dx}(1) = f'(1) = ?$$

- A) -3 B) -1 C) 0 D) 1 E) 2

(YÖS 2002)

Çözüm / Solution:

$$\frac{df}{dx} = f'(x) = \frac{1}{2\sqrt{x}} \cdot (x^3 - 3) + \sqrt{x} \cdot 3x^2$$

$$f'(1) = -1 + 3 = 2$$

Yanıt / Answer E

$$12. f(x) = \ln(\cos x) \Rightarrow \frac{df}{dx}\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right) = ?$$

- A) 0 B) -1 C) 1 D) $-e$ E) e

(YÖS 2002)

Çözüm / Solution:

$$\frac{df}{dx} = f'(x) = \frac{1}{\cos x} \cdot (\cos x)' = \frac{-\sin x}{\cos x} = -\tan x$$

$$f'\left(\frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$$

Yanıt / Answer B

$$13. f(x) = \ln(\sqrt{x^3 - 4}) \left(f'(x) = \frac{df(x)}{dx} \right) \Rightarrow f'(2) = ?$$

- A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) 0 D) 1 E) 2

(YÖS 2003)

Çözüm / Solution:

$$f'(x) = \frac{3x^2}{2\sqrt{x^3-4}}$$

$$f'(2) = \frac{12}{4} = \frac{3}{2}$$

Yanıt / Answer A

$$14. f(x) = (x-2)^4, \left(f''(x) = \frac{d^2 f(x)}{dx^2} \right) \Rightarrow f''(3) = ?$$

- A) 4 B) 6 C) 8 D) 10 E) 12

(YÖS 2003)

Çözüm / Solution:

$$f'(x) = 4(x-2)^3$$

$$f''(x) = 12(x-2)^2$$

$$f''(3) = 12(3-2)^2 = 12$$

Yanıt / Answer E

15. $f(x) = x \cdot |9 - x^2| \Rightarrow f'(4) = ?$

- A) 39 B) 28 C) 7 D) -7 E) -39

(YÖS 2004)

Çözüm / Solution:

$$\begin{aligned} f(x) &= x \cdot |9 - x^2| \\ \Rightarrow f'(x) &= 1 \cdot |9 - x^2| + x \cdot \frac{(-2x) \cdot |9 - x^2|}{9 - x^2} \\ &= 7 + \frac{4 \cdot (-8) \cdot 7}{-7} = 39 \end{aligned}$$

Yanıt / Answer A

16. $f(x) = \ln(\cos e^x) \Rightarrow f\left(\ln \frac{\pi}{4}\right) = ?$

- A) $-\frac{\pi}{4}$ B) $-\frac{\pi}{2}$ C) $\frac{\pi}{4}$
D) $e^{\frac{\pi}{4}}$ E) $e^{\frac{\pi}{2}}$

(YÖS 2004)

Çözüm / Solution:

$$\begin{aligned} f(x) &= \ln(\cos e^x) \\ \Rightarrow f'(x) &= \frac{-\sin e^x \cdot (e^x)'}{\cos e^x} \\ &= (-\tan e^x) \cdot (e^x)' \\ &= (-\tan e^{\ln \frac{\pi}{4}}) \cdot (e^{\ln \frac{\pi}{4}})' \\ &= -(\tan \frac{\pi}{4}) \cdot \frac{\pi}{4} \\ &= -\frac{\pi}{4} \end{aligned}$$

Yanıt / Answer A

17. $f(x) = x^3 + 2x^2 + \ln x - 1$

$$\left. \frac{d^2 f}{dx^2} \right|_{x=1} = f''(1) = ?$$

- A) 1 B) 4 C) 6 D) 8 E) 9

(YÖS 2005)

Çözüm / Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 4x + \frac{1}{x} \\ f''(x) &= 6x + 4 - \frac{1}{x^2} \\ f''(1) &= 6 + 4 - 1 = 9 \end{aligned}$$

Yanıt / Answer E

18. $f(x) = \sin(\ln x)$

$$\left. \frac{df}{dx} \right|_{x=e^\pi} = f'(e^\pi) = ?$$

- A) $-\frac{1}{e^\pi}$ B) $-\frac{2}{e^\pi}$ C) $\frac{1}{e^\pi}$
D) $\frac{e^\pi}{2}$ E) $2e^\pi$

(YÖS 2005)

Çözüm / Solution:

$$\begin{aligned} f'(x) &= \frac{1}{x} \cos(\ln x) \\ f'(e^\pi) &= \frac{1}{e^\pi} \cdot \cos(\ln e^\pi) \\ \frac{\cos \pi}{e^\pi} &= -\frac{1}{e^\pi} \end{aligned}$$

Yanıt / Answer A

19. $x = \ln t$
 $y = t^3 - 3t^2 - 3t$
 $\left. \frac{dy}{dx} \right|_{x=0} = ?$

- A) -6 B) -3 C) 1 D) 3 E) 9

(YÖS 2005)

Çözüm / Solution:

$$\frac{dy}{dx} = \frac{3t^2 - 6t - 3}{\frac{1}{t}}$$

$$x = \ln t \Rightarrow 0 = \ln t \Rightarrow t = 1$$

$$= \frac{3 - 6 - 3}{\frac{1}{1}} = -6$$

Yanıt / Answer A

20. $f(x) = x^2 - 1$
 $g(x) = x^3$
 $(g \circ f)'(x) = ?$

- A) $6x$ B) $x^3(x^2 - 1)$ C) $2x(x^2 - 1)^3$
D) $3x(x^2 - 1)^2$ E) $6x(x^2 - 1)^2$

(YÖS 2006)

Çözüm / Solution:

$$f(x) = x^2 - 1$$

$$g(x) = x^3$$

$$(g \circ f)'(x) = f'(x) \cdot g'(f(x))$$

$$= 2x \cdot 3 \cdot (x^2 - 1)^2$$

$$= 6x(x^2 - 1)^2$$

Yanıt / Answer E

21. $f(x) = \ln^2(2x^2 - 6) \Rightarrow f'(3) = ?$

- A) 0 B) $\ln 6$ C) $\ln 12$
D) $2\ln 6$ E) $2\ln 12$

(YÖS 2006)

Çözüm / Solution:

$$f(x) = \ln^2(2x^2 - 6)$$

$$f'(x) = 2\ln(2x^2 - 6) \cdot \frac{4x}{2x^2 - 6}$$

$$f'(3) = 2 \cdot \ln 12 \cdot \frac{12}{12} = 2\ln 12$$

Yanıt / Answer E

22. $f(x) = \arcsin x$

$$\left(f^{-1} \right)' \left(\frac{\pi}{4} \right) = ?$$

- A) -1 B) 0 C) $\frac{\sqrt{2}}{2}$ D) $\frac{3\sqrt{2}}{2}$ E) $3\sqrt{2}$

(YÖS 2006)

Çözüm / Solution:

$$f(x) = \arcsin x$$

$$f^{-1}(x) = \sin x$$

$$\Rightarrow (f^{-1})'(x) = \cos x$$

$$x = \frac{\pi}{4} \Rightarrow \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Yanıt / Answer C

23. $f(x) = \sin^3 x$

$\Rightarrow f\left(\frac{\pi}{3}\right) = ?$

A) $\frac{\sqrt{3}}{2}$

B) $\frac{\sqrt{3}}{4}$

C) $\frac{3}{4}$

D) $\frac{3}{8}$

E) $\frac{9}{8}$

(YÖS 2007)

Çözüm / Solution:

$f(x) = \sin^3 x$

$\Rightarrow f'(x) = 3 \cdot \sin^2 x \cdot \cos x$

$f'\left(\frac{\pi}{3}\right) = 3 \cdot \sin^2 \frac{\pi}{3} \cdot \cos \frac{\pi}{3}$

$= 3 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \frac{1}{2} = \frac{9}{8}$

Yanıt / Answer E

24. $f(x) = \arctan \frac{1}{x^2}$

$\Rightarrow f'(1) = ?$

A) -2

B) -1

C) 0

D) 1

E) 2

(YÖS 2007)

Çözüm / Solution:

$f(x) = \arctan \frac{1}{x^2}$

$\Rightarrow f'(x) = \frac{-\frac{2}{x^3}}{1 + \left(\frac{1}{x^2}\right)^2} \Rightarrow f'(1) = \frac{-\frac{2}{1}}{1 + (1)^2} = -\frac{2}{2}$

Yanıt / Answer B

1. $y = (1 - x^2)^3 \Rightarrow \left. \frac{d^2 y}{dx^2} \right|_{x=1} = ?$

Yanıt / Answer : 0

2. $f(x) = \sqrt{1+x^3} \Rightarrow f'(2) = ?$

Yanıt / Answer : 2

3. $x = f(t) = t^3 - 1$
 $y = g(t) = 2t^2 + 2t$
 $\Rightarrow \left. \frac{dy}{dx} \right|_{t=1} = ?$

Yanıt / Answer : 2

4. $f(x) = (x^2 + 1) \cdot g(2x + 1)$
 $g(5) = 6, g'(5) = 3$
 $\Rightarrow f'(2) = ?$

Yanıt / Answer : 54

5. $f(x) = (x^2)^{\sin x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

Yanıt / Answer : π

6. $f(x) = x(2 - \ln x) \Rightarrow \frac{df(e)}{dx} = ?$

Yanıt / Answer : 0

7. $f(x) = \frac{\ln x}{x^2 + 1} \Rightarrow \frac{df(1)}{dx} = f'(1) = ?$

Yanıt / Answer : $\frac{1}{2}$

8. $y = \frac{x}{(x-1)^2} \Rightarrow (x-1)^4 \cdot \frac{dy}{dx} = ?$

Yanıt / Answer : $1 - x^2$

9. $f(x) = \frac{ax^2+1}{x^2+1}$, $\frac{d}{dx} f(2) = 4 \Rightarrow a = ?$

Yanıt / Answer : 26

10. $f(x) = \sin 6x \cdot \cos 6x \Rightarrow f'(\frac{\pi}{12}) = ?$

Yanıt / Answer : -6

11. $\lim_{x \rightarrow \pi} (x - \pi) \cdot \operatorname{cosec} x = ?$

Yanıt / Answer : -1

12. $y = \sin\left(\frac{2x-1}{x+1}\right)$
 $x = \frac{\pi+2}{4-\pi}$ $\left\{ \Rightarrow \frac{dy}{dx} = ? \right.$

Yanıt / Answer : 0

13. $f(x) = (x^2 + 1) \cdot \ln(2x + 1) \Rightarrow f'(1) = ?$

Yanıt / Answer : $\frac{2\ln(27 \cdot e^2)}{3}$

14. $\left. \begin{array}{l} x = t^3 + 3t \\ y = t^3 - 3t \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} \Big|_{t=1} = ?$

Yanıt / Answer : $\frac{1}{6}$

15. $f(x) = x^{2x} \Rightarrow f'(2) = ?$

Yanıt / Answer : $32(\ln 2 + 1)$

16. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = ?$

Yanıt / Answer : 0

$$17. \quad y = \sqrt{4 + \ln x} \Rightarrow \frac{dy}{dx} = ?$$

$$\text{Yanıt / Answer : } \frac{1}{2x\sqrt{4 + \ln x}}$$

$$18. \quad x^3 + 4xy^2 - y^4 - 27 = 0 \Rightarrow \frac{dy}{dx} = ?$$

$$\text{Yanıt / Answer : } \frac{3x^2 + 4y^2}{4y^3 - 8xy}$$

$$19. \quad y = x^x \Rightarrow \frac{dy}{dx} = ?$$

$$\text{Yanıt / Answer : } x^x \cdot \ln(x)$$

$$20. \quad y = e^x \cdot x^{3x} \Rightarrow \frac{dy}{dx} = ?$$

$$\text{Yanıt / Answer : } e^x \cdot x^{3x} \cdot (4 + 3 \ln x)$$

$$21. \quad y^2 = e^{x+y} \Rightarrow \frac{d^2 y}{dx^2} = ?$$

$$\text{Yanıt / Answer : } \frac{2y}{(2-y)^2}$$

$$22. \quad f(x) = \frac{\sqrt{1+2x} - (1+x)}{x^2} \Rightarrow \lim_{x \rightarrow 0} f(x) = ?$$

$$\text{Yanıt / Answer : } -\frac{1}{2}$$

$$23. \quad f(t) = \frac{\sqrt[3]{1+3t} - (1+t)}{t^2} \Rightarrow \lim_{t \rightarrow 0} \frac{\sqrt[3]{1+3t} - (1+t)}{t^2} = ?$$

$$\text{Yanıt / Answer : } -1$$

$$24. \quad f(x) = \frac{2 \sin^2 x}{1 - \cos x} \Rightarrow \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = ?$$

$$\text{Yanıt / Answer : } 4$$

25. $\lim_{x \rightarrow \pi} \frac{\cos \frac{x}{2}}{x - \pi} = ?$

Yanıt / Answer : $-\frac{1}{2}$

26. $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin^2 x} = ?$

Yanıt / Answer : $-\frac{1}{2}$

27. $\lim_{x \rightarrow 0} \sin 4x \cdot \cot 2x = ?$

Yanıt / Answer : 2

28. $\lim_{x \rightarrow 0} \frac{4 \sin 9x}{3x} = ?$

Yanıt / Answer : 12

29. $\left. \begin{matrix} y = t^3 + t \\ x = t^3 + 1 \end{matrix} \right\} \Rightarrow \frac{d^2 y}{dx^2} \Big|_{x=2} = ?$

Yanıt / Answer : $-\frac{2}{9}$

30. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos^2 x - \sin^2 x} = ?$

A) $-\sqrt{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $-\frac{1}{2}$ D) $\frac{\sqrt{2}}{2}$ E) $\sqrt{2}$

Yanıt / Answer : $-\frac{\sqrt{2}}{2}$

31. $\lim_{x \rightarrow 2} \frac{\ln x - \ln 2}{x^2 - 4} = ?$

Yanıt / Answer : $\frac{1}{8}$

32. $e^{-x^3} \frac{d^2}{dx^2} (x^2 e^{x^3}) = ?$

Yanıt / Answer : $9x^6 + 18x^3 + 2$

1. $f(x) = x^2 + \sqrt{x} \Rightarrow f'(9) = ?$

- A) 18 B) $\frac{109}{6}$ C) $\frac{55}{3}$ D) $\frac{37}{2}$ E) $\frac{56}{3}$

2. $f(x) = \frac{x}{x^2-1} \Rightarrow f'(2) = ?$

- A) $-\frac{7}{9}$ B) $\frac{2}{3}$ C) $-\frac{5}{9}$ D) $-\frac{4}{9}$ E) $-\frac{1}{3}$

3. $\left. \begin{array}{l} x = \ln t \\ y = t^2 \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} = ?$

- A) t B) t^2 C) $2t^2$ D) $4t^2$ E) $8t^2$

4. $f(x) = \ln(\cos x + \sin x) \Rightarrow f'(x) = ?$

- A) $\frac{\cos 2x}{\sin x + \cos x}$ B) $\frac{\sin x \cdot \cos x}{\sin x + \cos x}$
 C) $\frac{\sin x - \cos x}{\cos x + \sin x}$ D) $\frac{\cos x - 2 \sin x}{\cos x + \sin x}$
 E) $\frac{\cos x - \sin x}{\cos x + \sin x}$

5. $x^2 - xy - y^2 + 5 = 0 \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{y-x}{x+2y}$ B) $\frac{y-2x}{x+2y}$ C) $\frac{y+2x}{x-2y}$
 D) $\frac{2x-y}{x+2y}$ E) $\frac{y+x}{x-2y}$

6. $f(x) = (3-2x)^6 \Rightarrow f'(1) = ?$

- A) -12 B) -6 C) -3 D) 3 E) 6

7. $y = \sin^3 x \Rightarrow \frac{dy}{dx} = ?$

- A) $3 \sin^3 x \cdot \cos x$ B) $3 \sin^2 x \cdot \cos x$
 C) $3 \sin x \cdot \cos x$ D) $3 \sin^2 x \cdot \tan x$
 E) $3 \sin x \cdot \cot x$

8. $f(x) = \cos(\sin x) \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

- A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

9. $f(x) = \ln \frac{x-a}{x+a} \Rightarrow f'(x) = ?$

- A) $\frac{2a+2x}{x^2-a^2}$ B) $\frac{2x^2}{a^2-x^2}$ C) $\frac{2x^2}{x^2-a^2}$
 D) $\frac{2a}{a^2-x^2}$ E) $\frac{2a}{x^2-a^2}$

10. $f(x) = e^{\sin x} \Rightarrow f'(0) = ?$

- A) $-e^2$ B) $-e$ C) 1 D) e E) e^2

11. $\frac{1}{2 \cdot \cos 2x} \cdot \frac{d}{dx} (\sin^2 x) = ?$

- A) $\frac{\cot 2x}{\tan x}$ B) $\frac{\cot x}{2}$ C) $\frac{\cot 2x}{2}$
D) $\frac{\tan 2x}{2}$ E) $\frac{\tan x}{2}$

12. $f(x) = 3^{\cos x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

- A) $-\ln 6$ B) $-\ln 3$ C) $\ln 2$
D) $\ln 3$ E) $\ln 6$

13. $\sin(xy) = x^2 + y^3 \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{2x - y \cdot \cos(xy)}{x \cdot \cos(xy) - 3y^2}$ B) $\frac{2x + y \cdot \sin(xy)}{x \cdot \cos(xy) - 3y^2}$
C) $\frac{2x + y \cdot \cos(xy)}{x \cdot \sin(xy) - 3y^2}$ D) $\frac{2x - y \cdot \cos(xy)}{x \cdot \cos(xy) + 3y^2}$
E) $\frac{2x + y \cdot \cos(xy)}{x \cdot \cos(xy) + 3y^2}$

14. $\begin{cases} x = 2 \sin t \\ y = 3 \cos t \end{cases} \Rightarrow \frac{d^2 y}{dx^2} = ?$

- A) $\frac{-3}{\cos^3 t}$ B) $\frac{-3}{4 \sin t}$ C) $\frac{3}{4 \tan t}$
D) $\frac{3}{4 \sin t}$ E) $\frac{3 \sin t}{4 \cos t}$

15. $f(x) = 5^x - 8^x \Rightarrow f'(0) = ?$

- A) $\ln \frac{3}{8}$ B) $\ln \frac{1}{2}$ C) $\ln \frac{5}{8}$
D) $\ln \frac{3}{4}$ E) $\ln \frac{7}{8}$

16. $f(x) = \sqrt[5]{-x^3 + 2x} \Rightarrow f'(-1) = ?$

- A) -5 B) $-\frac{1}{5}$ C) 1 D) $\frac{1}{5}$ E) 5

17. $f(x) = \ln(\cos x) \Rightarrow f'\left(\frac{\pi}{3}\right) = ?$

- A) $-2\sqrt{3}$ B) $-\sqrt{3}$ C) -1
D) $\sqrt{3}$ E) $2\sqrt{3}$

18. $y = \log_2 x^2 \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{2+x}{\ln 2}$ B) $\frac{2}{x \cdot \ln 2}$ C) $\frac{1}{x \ln 2}$
D) $\frac{x \cdot \ln 2}{x + \ln 2}$ E) $\frac{\ln 2}{x + \ln 2}$

Yanıtlar / Answers

1. B	2. C	3. D	4. E	5. D	6. A
7. B	8. C	9. E	10. C	11. D	12. B
13. A	14. A	15. C	16. B	17. B	18. B

1. $f(x) = 2x^3 - 3x^2 - 12x + 20 \Rightarrow f'(-1) + f''(1) = ?$

- A) -6 B) -1 C) 0 D) 1 E) 6

2. $f(x) = x^3 + ax^2 + bx + 3,$
 $f'(1) = -2, f''(2) = 0 \Rightarrow b = ?$

- A) 2 B) 3 C) 5 D) 6 E) 7

3. $y = \frac{2x+1}{x-2} \Rightarrow \frac{dy}{dx} \Big|_{x=3} = ?$

- A) -5 B) -4 C) -3 D) 3 E) 4

4. $f(x) = (2x-1)^3 \cdot \ln x \Rightarrow f'(1) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

5. $f(x) = x \cdot e^x \Rightarrow f'(2) + f''(2) = ?$

- A) $6e^2$ B) $7e^2$ C) $\frac{1}{6e^2}$ D) $\frac{1}{7e^2}$ E) $\frac{6}{e^2}$

6. $f(x) = \sqrt{e^x} \cdot \ln(x^2) \Rightarrow f'(1) = ?$

- A) $\frac{2}{\sqrt{e}}$ B) $\frac{4}{\sqrt{e}}$ C) $2\sqrt{e}$ D) $3\sqrt{e}$ E) $4\sqrt{e}$

7. $f(3x-2) = x^3 - 3x + 1 \Rightarrow f'(4) = ?$

- A) 2 B) 3 C) 4 D) 5 E) 6

8. $\left. \begin{array}{l} f(x) = \sqrt{x} \\ g(x) = x^2 + 3 \end{array} \right\} \Rightarrow (f \circ g)'(1) = ?$

- A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

9. $f(x) = \cos(2x)$

$\Rightarrow \lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{6} + h\right) - f\left(\frac{\pi}{6}\right)}{h} = ?$

- A) $-\sqrt{3}$ B) $-\frac{1}{\sqrt{3}}$ C) 1 D) 2 E) $\sqrt{3}$

10. $f(x) = \arccos(2x+1) \Rightarrow f'\left(-\frac{1}{2}\right) = ?$

- A) -2 B) -1 C) 0 D) $\frac{1}{2}$ E) $-\frac{1}{2}$

11. $f(x) = \tan(\sqrt[3]{x}) \Rightarrow f'(\pi^3) = ?$

- A) π^2 B) $2\pi^2$ C) $3\pi^2$ D) $\frac{1}{3\pi^2}$ E) $\frac{1}{2\pi^2}$

12. $f(x) = \sin^2 \sqrt{x} \Rightarrow \frac{df(x)}{dx} = f'(x) = ?$

- A) $\frac{1}{2\sqrt{x}} \sin 2\sqrt{x}$ B) $\frac{1}{\sqrt{x}} \sin 2\sqrt{x}$ C) $\frac{1}{2\sqrt{x}} \sin \sqrt{x}$
D) $\frac{1}{2\sqrt{x}} \sin 2\sqrt{x} \cdot \cos \sqrt{x}$ E) $\frac{1}{4\sqrt{x}} \sin 2\sqrt{x}$

13. $f(x) = \ln(\arctan x) \Rightarrow f'(1) = ?$

- A) $\frac{\pi}{3}$ B) $\frac{2}{\pi}$ C) $\frac{\pi}{4}$ D) $\frac{1}{2\pi}$ E) π

14. $f(x) = \log x^3 \Rightarrow f\left(\frac{1}{\ln 10}\right) = ?$

- A) 3 B) 1 C) 0 D) $3\ln 10$ E) $(\ln 10)^2$

15. $f(x) = 5^{3x-3}, f(a) = \ln 5^{375} \Rightarrow a = ?$

- A) 4 B) 3 C) 0 D) 1 E) 2

16. $\left. \begin{array}{l} x = t^3 - 2t^2 + 3t \\ y = t^3 - 2t \end{array} \right\} \Rightarrow \frac{dy}{dx} \Big|_{t=1} = ?$

- A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

17. $\left. \begin{array}{l} y = e^x \\ x = \cos t \end{array} \right\} \Rightarrow \frac{dy}{dt} \Big|_{t=\frac{\pi}{3}} = ?$

- A) $-\frac{\sqrt{3}e}{2}$ B) $-\frac{\sqrt{e}}{4}$ C) $-\frac{\sqrt{2e}}{3}$ D) $\sqrt{3e}$ E) $\sqrt{2e}$

18. $\left. \begin{array}{l} y = e^t + 2t \\ x = e^{2t} \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} \Big|_{t=0} = ?$

- A) $-\frac{3}{4}$ B) $-\frac{5}{4}$ C) $-\frac{7}{4}$ D) $\frac{1}{2}$ E) $\frac{3}{2}$

19. $2x^2 + 3xy - 4y^2 = 0 \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{4x-3y}{3x+8y}$ B) $\frac{-4x-3y}{3x-8y}$ C) $\frac{4x+3y}{3x+8y}$
D) $\frac{2x-3y}{3x+4y}$ E) $\frac{2x+3y}{3x-8y}$

Yanıtlar / Answers

1. E	2. E	3. A	4. D	5. B	6. C
7. B	8. B	9. A	10. A	11. D	12. A
13. B	14. A	15. E	16. B	17. A	18. A
19. B					

1. $f(x) = ax^2 - 3x + 1$

$$\frac{d}{dx} f(1) = 5 \Rightarrow a = ?$$

- A) 6 B) 5 C) 4 D) 3 E) 2

2. $f(x) = x^4 - 3x^2 + 6x + 3$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = ?$$

- A) 2 B) 4 C) 5 D) 10 E) 15

3. $f(x) = ax^2 - bx$

$$\Rightarrow \frac{d}{dx} f(1) = ?$$

- A)
- $6a - b$
- B)
- $3a + 2$
- C)
- $2a + b$
-
- D)
- $3a - 2b$
- E)
- $2a - b$

4. $a < 0,$

$$f(x) = \frac{1}{3}x^3 + x^2 - 3x + 7$$

$$\frac{d}{dx} f(a) = 0 \Rightarrow a = ?$$

- A) -4 B) -3 C) -2 D) 1 E) 6

5. $f(x) = x^3 - bx^2 + 6x + 3$

$$\frac{d}{dx} f(2) = \frac{d}{dx} f(1)$$

$$\Rightarrow b = ?$$

- A)
- $\frac{7}{2}$
- B)
- $\frac{9}{2}$
- C)
- $\frac{11}{2}$
- D)
- $\frac{15}{2}$
- E)
- $\frac{19}{2}$

6. $f(x) = x^3 - ax^2 + bx + 3,$
 $f(1) = 12; f'(2) = 8$ } $\Rightarrow b = ?$

- A) 4 B) 6 C) 8 D) 12 E) 16

7. $\forall x \in \mathbb{N}^+, f(0) = 0$

$$f(x) - f(x-1) = x + 1$$

$$\Rightarrow f'(6) = ?$$

- A)
- $\frac{7}{2}$
- B)
- $\frac{9}{2}$
- C)
- $\frac{11}{2}$
- D)
- $\frac{13}{2}$
- E)
- $\frac{15}{2}$

8. $\forall x \in \mathbb{Z}^+, f(0) = 0$

$$f(x) = x^2 + f(x-1)$$

$$\Rightarrow f'(1) + f'(0) = ?$$

- A)
- $\frac{7}{3}$
- B)
- $\frac{4}{3}$
- C) 1 D)
- $\frac{8}{3}$
- E) 3

9. $\forall f'(1) + f'(0) \in \mathbb{R}, f(x) = f(-x)$

$f(x) = 4x^4 - 2ax^2 + 4 - f(-x)$

$\frac{d}{dx} f(2) = 32 \Rightarrow a = ?$

- A) 2 B) 4 C) 5 D) 6 E) 8

10. $\forall x \in \mathbb{R}, f(x) + f(-x) = 0$

$2f(x) = 5x^3 - 10ax - 20 + 3f(-x)$

$\frac{d}{dx} f(3) = 15 \Rightarrow a = ?$

- A) 2 B) 3 C) 4 D) 5 E) 6

11. $f(2x-1) = 2x^2 - 6x + 4$

$\Rightarrow \frac{df(x)}{dx} = ?$

- A) $2x + 1$ B) $2x - 1$ C) $x - 2$
D) $2x - 2$ E) $x - 3$

12. $f(2x-1) = x^3 - x^2 + 4x + 1$

$\Rightarrow f(3) + f'(3) = ?$

- A) 15 B) 17 C) 19 D) 21 E) 23

13. $f(3x-4) = x^3 - 6x^2 + 7$

$\Rightarrow f(2) + f'(2) = ?$

- A) -15 B) -14 C) -13 D) -12 E) -10

14. $f(x) = 2x - 3$

$(g \circ f)(x) = 4x^2 - 8x + 7$

$\Rightarrow \frac{dg(x)}{dx} = ?$

- A) $x + 2$ B) $2(x + 1)$ C) $2(x + 2)$
D) $x - 2$ E) $2(x - 2)$

15. $f(x^3 + 2) = 3x^9 - 6x^6 + 5$

$\Rightarrow \frac{df(x)}{dx} = ?$

- A) $9x^2 - 32x$ B) $9x^2 - 36x + 40$
C) $9x^2 - 48x + 60$ D) $9x^2 - 42x + 50$
E) $9x^2 - 18x - 70$

16. $f(x) = (ax^2 - 1)(x^2 + 2x + 3)$

$\frac{d}{dx} f(2) = 130$

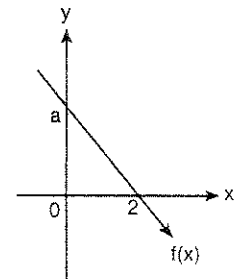
$\Rightarrow a = ?$

- A) 2 B) 3 C) 5 D) 13 E) 21

17. $g(x) = x^2 \cdot f(x)$

$g'(4) = -48$

$\Rightarrow a = ?$



- A) 2 B) 3 C) 4 D) 5 E) 6

Yanıtlar / Answers

1. C	2. B	3. E	4. B	5. B	6. D
7. E	8. A	9. E	10. E	11. C	12. C
13. C	14. B	15. C	16. A	17. B	

1. $y = \sqrt[7]{x^2}$

$$\Rightarrow \sqrt[7]{x^5} \cdot \frac{dy}{dx} = ?$$

- A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{1}{\sqrt{x}}$ D) $\frac{3}{7}$ E) $\frac{x}{\sqrt{x}}$

2. $f(x) = \sqrt[3]{x^2}$ ($a \neq 0$)

$$\frac{df(a)}{dx} = f(a) \Rightarrow a = ?$$

- A) 4 B) 2 C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) -1

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3. $a < 0$

$$f(x) = \sqrt[3]{x^2 + a}$$

$$f'(1) = \frac{1}{6} \Rightarrow a = ?$$

- A) -9 B) -7 C) -5 D) -3 E) -1

4. $f(x) = \sqrt[3]{x+a}$

$$f^{-1}(1) + (f^{-1})'(1) = 2 \Rightarrow a = ?$$

- A) 1 B) 2 C) 3 D) 4 E) 5

5. $f(x) = \frac{(x-2)^3}{2}$

$$\Rightarrow (f^{-1})(4) + (f^{-1})'(4) = ?$$

- A) $\frac{16}{17}$ B) $\frac{1}{25}$ C) $\frac{5}{3}$ D) $\frac{25}{6}$ E) 25

6. $f(x) = \frac{(x-1)^3}{4}$

$$(f^{-1})(16) \cdot (f^{-1})'(16) = ?$$

- A) $\frac{5}{4}$ B) $\frac{5}{8}$ C) $\frac{5}{12}$ D) $\frac{5}{16}$ E) $\frac{5}{48}$

7. $y = \frac{x}{3\sqrt{x}} \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{1}{3\sqrt{x}}$ B) $\frac{1}{2\sqrt{x}}$ C) $\frac{1}{4\sqrt{x}}$
D) $\frac{1}{6\sqrt{x}}$ E) $\frac{1}{9\sqrt{x}}$

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8. $f(x) = \sqrt[3]{\frac{x}{16}}$

$$\Rightarrow f(2) \cdot (f^{-1})'(1) = ?$$

- A) 12 B) 18 C) 20 D) 24 E) 28

9. $f(x) = (x+1)^2 \cdot (3x+1)^2$

$$\Rightarrow \frac{df(1)}{dx} = ?$$

- A) 96 B) 120 C) 144
D) 156 E) 160

10. $f(x) = \frac{\sqrt{x}}{1+\sqrt{x}} \Rightarrow \frac{df(4)}{dx} = ?$

- A) 12 B) 1 C) $\frac{1}{36}$ D) $\frac{1}{16}$ E) $\frac{1}{12}$

11. $y = (x^2 + 1) \sqrt{x} \Rightarrow \frac{dy}{dx} = y' = ?$

A) $\frac{x^2 + 1}{2\sqrt{x}}$

B) $\frac{2x^2 + 1}{2\sqrt{x}}$

C) $\frac{5x^2 + 1}{2\sqrt{x}}$

D) $\frac{3x^2 + 2}{2\sqrt{x}}$

E) $\frac{6x^2 + 3}{2\sqrt{x}}$

12. $f(x) = \log_2 x$

$\Rightarrow \frac{d}{dx} f(1) = ?$

A) $\log e$

B) $2 \log e$

C) $\log_2 e$

D) $\frac{2}{\log e}$

E) $\frac{2}{10} \log e$

13. $a > 0, b > 0$

$f(x) = \ln(ax + 6)$

$\frac{d}{dx} f(1) = \frac{a}{5} \Rightarrow a = ?$

A) -2

B) -1

C) 0

D) 1

E) 2

14. $a > 2$

$f(x) = a \cdot \ln(ax - 3)^2$

$\frac{d}{dx} f(2) = 8$

$\Rightarrow a = ?$

A) 1

B) 3

C) 4

D) 5

E) 6

15. $f(x) = \log_a(x^2 + 3x + 1)$

$\frac{d}{dx} f(1) = \frac{1}{6}$

$\Rightarrow a = ?$

A) e^3

B) e^4

C) e^5

D) e^6

E) e^{30}

16. $f(x) = x \cdot \ln x$

$\Rightarrow \frac{d}{dx} f(e) = ?$

A) $\frac{1}{2}$

B) 1

C) 2

D) $\frac{3}{2}$

E) 3

17. $f(x) = x^2 \cdot \ln(x^2 + 1)$

$\Rightarrow \frac{d}{dx} f(1) = ?$

A) $\ln 2e$

B) $\ln \frac{2}{e}$

C) $\ln 4e$

D) $\ln \frac{4}{e}$

E) $\ln 3e$

18. $y = \ln \left(\frac{x^2}{x^2 + 1} \right)$

$\Rightarrow (x^2 + 1) \cdot \frac{dy}{dx} = ?$

A) $2x^2 + 1$

B) $2x^2 + 2$

C) $\frac{2}{x^2}$

D) $\frac{2}{x}$

E) $4x^2 + 2$

19. $x > 0, f(x) = \ln \sqrt[k]{x^2}$

$\frac{df(4)}{dx} = \frac{1}{10} \Rightarrow k = ?$

A) 5

B) 8

C) 10

D) 12

E) 18

Yanıtlar / Answers

1. B	2. C	3. A	4. B	5. D	6. C
7. D	8. D	9. E	10. C	11. C	12. A
13. B	14. E	15. D	16. C	17. C	18. D
19. A					

1. $f(x) = (3x^2 - 2x + 1)^4 \Rightarrow f'(-1) = ?$

- A) -6912 B) -6180 C) -5900 D) 2300 E) 1800

2. $f(x) = x\sqrt{x} - x\sqrt{1-x} \Rightarrow f'\left(\frac{3}{4}\right) = ?$

- A) $\frac{3\sqrt{3}+1}{4}$ B) $\frac{3\sqrt{3}+4}{4}$ C) $\frac{3\sqrt{3}+2}{4}$
D) $3\sqrt{3}-1$ E) $\frac{5\sqrt{3}+1}{4}$

3. $f(x) = 3\sin(3x + 5) \Rightarrow f'(7^\circ) = ?$

- A) $3 \cos 26$ B) $9 \cos 26$ C) $15 \cos 26$
D) $3 \sin 26$ E) $-\sin 26$

4. $f(x) = \frac{x \cdot \sin x}{1 + \tan x} \Rightarrow f'\left(\frac{\pi}{4}\right) = ?$

- A) $\sqrt{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{2}}{4}$ D) $-\sqrt{2}$ E) -2

5. $f(x) = \arccos \sqrt{1-4x} \Rightarrow f'(x) = ?$

- A) $-2\sqrt{4x^2}$ B) $-\sqrt{x+4x^2}$ C) $\frac{-2}{\sqrt{x-4x^2}}$
D) $\frac{1}{\sqrt{x-4x^2}}$ E) $-\sqrt{x+4x^2}$

6. $f(x) = \frac{2}{3} \arctan x + \frac{1}{3} \arctan \frac{x}{1-x^2} \Rightarrow f'(1) = ?$

- A) -1 B) 0 C) $\frac{1}{2}$
D) $\frac{3}{4}$ E) 1

7. $f(x) = (x+1)^{\frac{1}{x}} \Rightarrow f'(1) = ?$

- A) $\ln \frac{e}{4}$ B) $\ln 2e$ C) $2 - \ln 2$
D) 1 E) -1

8. $y = f(x), \frac{x-y}{x-2y} = 2 \Rightarrow f'(x) = ?$

- A) $\frac{2}{3}$ B) 1 C) $\frac{1}{3}$
D) $-\frac{1}{3}$ E) $-\frac{4}{3}$

9. $f(3x) = x^2, g(x-2), f'(3) = 2, g(-1) = 4 \Rightarrow g'(-1) = ?$

- A) 3 B) 2 C) -2 D) -3 E) -4

10. $f(x) = e^{\sin x} + \cos^2 x \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

- A) -1 B) 0 C) 1 D) 2 E) e

11. $f(x) = \sin x$, $g(x) = 2^x \Rightarrow (f \circ g)'(2) = ?$

- A) $\ln 2 \cdot \cos 16$
 B) $\ln 4 \cdot \cos 4$
 C) $\ln 8 \cdot \cos 4$
 D) $\ln 16 \cdot \cos 4$
 E) $\ln 16 \cdot \cos 16$

12. $\left. \begin{array}{l} x = e^t + t^2 \\ y = t \cdot e^t \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} \Big|_{t=1} = ?$

- A) $\frac{1}{e^2}$ B) $\frac{1}{2+e}$ C) $\frac{1}{4+e}$
 D) $\frac{e}{(e+2)^2}$ E) $\frac{e}{(e+2)^3}$

13. $\left. \begin{array}{l} x = t^2 - 2t \\ y = t^3 + t \end{array} \right\} \Rightarrow \frac{d^2y}{dx^2} \Big|_{t=1} = ?$

- A) 2 B) $\frac{4}{3}$ C) 1 D) $\frac{3}{4}$ E) $\frac{1}{4}$

14. $\left. \begin{array}{l} x = \ln t \\ y = \sin t \end{array} \right\} \Rightarrow \frac{dy}{dx} \Big|_{t=\pi} = ?$

- A) $-\pi$ B) -2π C) 0 D) 1 E) 2

15. $f(x) = x \cdot \sin x \Rightarrow \frac{d^2f(x)}{dx^2} \Big|_{x=\frac{\pi}{2}} = ?$

- A) -1 B) 0 C) 1 D) $\frac{\pi}{2}$ E) $-\frac{\pi}{2}$

16. $f(x) = x \cdot e^x$

$\Rightarrow f''(x) - f'(x) = ?$

- A) 0 B) x C) e^x D) xe^x E) $x + e^x$

17. $f(x) = \tan\left(\frac{\pi}{2} \cos x\right) \Rightarrow f'\left(\frac{\pi}{3}\right) = ?$

- A) $-\frac{\pi\sqrt{3}}{2}$ B) $-\frac{\pi}{2}$ C) 0
 D) π E) $\frac{\pi\sqrt{3}}{2}$

18. $f(x) = x \cdot \arcsin x + \sqrt{1-x^2} \Rightarrow f'\left(\frac{1}{2}\right) = ?$

- A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{2}$

19. $f(x) = (x^2 - x)^{-2} \Rightarrow f'(2) = ?$

- A) -4 B) $-\frac{8}{3}$ C) $-\frac{3}{4}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

Yanıtlar / Answers

1. A	2. A	3. B	4. C	5. D	6. E
7. A	8. C	9. C	10. B	11. D	12. D
13. E	14. A	15. E	16. C	17. A	18. B
19. C					

1. $f(x) = \frac{2x^3 + 1}{x} \Rightarrow f'(x) = ?$

- A) $2x - \frac{2}{x^3}$ B) $2 + 2x^2$ C) $2x^2 - \frac{1}{x^3}$
 D) $4x - \frac{1}{x^2}$ E) $x - \frac{1}{x^3}$

2. $f(x) = \frac{x^2 + 1}{x^2 - 1} \Rightarrow f'(x) = ?$

- A) $\frac{2x}{(x^2 - 1)^2}$ B) $\frac{4x}{(x^2 - 1)^2}$ C) $\frac{-4x}{(x^2 - 1)^2}$
 D) $\frac{-8x}{(x^2 - 1)^3}$ E) $\frac{-8x}{x^2 - 1}$

3. $f(x) = \frac{x^3}{3} - \frac{x^2}{2} + x - 1 \Rightarrow f'(2) = ?$

- A) 6 B) 5 C) 4 D) 3 E) 2

4. $y = (x^3 - 3x)^4 \Rightarrow \frac{dy}{dx} = ?$

- A) $4(x^2 - 3x)^4$ B) $3(x^2 - 3)(x^3 - 3x)^3$
 C) $12(x^2 - 1)(x^3 - 3x)^3$ D) $12(x^2 - 3)$
 E) $12(x^3 - 3x)^4$

5. $f(3x - 4) = (x^2 - 2)^3 \Rightarrow f'(2) = ?$

- A) 3 B) 4 C) 6 D) 7 E) 8

6. $f(3x - 2) = (9x^2 + 3x - 6)^2 \Rightarrow \frac{df(x)}{dx} = ?$

- A) $(2x + 5) \cdot (x^2 + 5x)$ B) $2(2x + 5)(x^2 + 5x)$
 C) $4(2x + 5)^2$ D) $2x(x^2 + 5x)$
 E) $4x(x^2 + 5x)$

7. $f(x) = \frac{(x+1)^2}{(x^2+1)^3} \Rightarrow f'(0) = ?$

- A) -1 B) 0 C) 1 D) $\frac{3}{2}$ E) 2

8. $f(x) = (x^2 + 1)^3(x^3 - 1)^2 \Rightarrow \frac{df(x)}{dx} = ?$

- A) $6x^2(x^2 + 1)(x^3 - 1) \cdot (2x^3 + x)$
 B) $6x(x^2 + 1)^2(x^3 - 1)^3(2x^3 + x - 2)$
 C) $36x(x^2 + 1)(x^3 - 1) \cdot (2x^3 + x - 2)$
 D) $6x(x^2 + 1)^2(x^3 - 1) \cdot (2x^3 + x - 1)$
 E) $12x(x^2 + 1)^2(x^3 - 1) \cdot (x^3 + x + 1)$

9. $x^3 + y^3 = 1 \Rightarrow \frac{d^2y}{dx^2} = ?$

- A) $\frac{-x}{y^3}$ B) $\frac{-2x}{y^5}$ C) $\frac{x}{y^4}$
 D) $\frac{4x}{y^5}$ E) $\frac{-4x}{y^5}$

10. $\left. \begin{aligned} x &= f(t) = t + \frac{1}{t} \\ y &= g(t) = t - \frac{1}{t} \end{aligned} \right\} \Rightarrow \frac{d^2y}{dx^2} = ?$

- A) $\frac{4t^3}{(t^2-1)^3}$ B) $\frac{-2t}{(t^2-1)}$ C) $\frac{-4t^2}{(t^2-1)^3}$
D) $\frac{-4t^3}{(t^2-1)^3}$ E) $\frac{8t}{(t^2+1)^3}$

11. $f(x) = x^2 + 1,$

$g(x) = \sqrt{x^2 + 1} \Rightarrow \frac{df(x)}{dg(x)} = ?$

- A) $2f(x)$ B) $\frac{2}{g(x)}$ C) $g(x)$
D) $2g(x)$ E) $\frac{g(x)}{f(x)}$

12. $f(x) = (3x-4) \cdot \sqrt[4]{(x+1)^3}$
 $\Rightarrow f'(x) = ?$

- A) $\frac{x}{12\sqrt[4]{(x+1)^3}}$ B) $\frac{12x-9}{4\sqrt[4]{x+1}}$ C) $\frac{21x}{2\sqrt[4]{(x+1)^3}}$
D) $\frac{7x}{4\sqrt[4]{x+1}}$ E) $\frac{35x}{6\sqrt[4]{x+1}}$

13. $f(x) = \frac{3x^2-1}{\sqrt[3]{(x^3-1)^2}} \Rightarrow \frac{df(x)}{dx} = ?$

- A) $\frac{2x(x-3)}{\sqrt[3]{x^3-1}}$ B) $\frac{x(x-3)}{(x^3-1)(\sqrt[3]{x^3-1})}$
C) $\frac{2x(x-3)}{(x^3-1)\sqrt[3]{(x^3-1)^2}}$ D) $\frac{x-3}{(x^3-1)^2\sqrt[3]{x^3-1}}$
E) $\frac{2x}{(x^3-1)\sqrt[3]{(x^3-1)}}$

14. $x^2 - xy + y^2 - 3 = 0 \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{2x-y}{x-2y}$ B) $\frac{2x+y}{x-2y}$ C) $\frac{x-2y}{2y+x}$
D) $\frac{x+y}{x-y}$ E) $\frac{y-2x}{x+2y}$

15. $\arctan \frac{x}{y} + \ln \sqrt{x^2 + y^2} = 0 \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{x-y}{x+y}$ B) $\frac{x+y}{x-y}$ C) $\frac{2x+y}{2x-y}$
D) $\frac{y-2x}{2x+y}$ E) $\frac{x+y}{x-2y}$

16. $f(x) = \arcsin \frac{x-1}{3} \Rightarrow f'(1) = ?$

- A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{3}$ C) $\frac{1}{3\sqrt{3}}$ D) $\frac{\sqrt{3}}{6}$ E) $\frac{1}{9}$

17. $f(x) = x \cdot \sin \frac{1}{x} \Rightarrow \frac{d^2f(x)}{dx^2} = ?$

- A) $\frac{1}{x^2} \cos \frac{1}{x}$ B) $-\frac{1}{x^3} \cos \frac{1}{x}$ C) $\frac{1}{x} \sin \frac{1}{x}$
D) $\frac{1}{x^3} \sin \frac{1}{x}$ E) $-\frac{1}{x^3} \sin \frac{1}{x}$

Yanıtlar / Answers					
1. D	2. C	3. D	4. C	5. E	6. B
7. E	8. D	9. B	10. D	11. D	12. B
13. C	14. A	15. B	16. B	17. E	

1. $f(x) = \frac{x^3+1}{x^3+3} \Rightarrow f'(3) = ?$

- A) $\frac{1}{25}$ B) $\frac{2}{25}$ C) $\frac{3}{50}$ D) $\frac{3}{20}$ E) $\frac{4}{75}$

2. $f(x) = \frac{1-\cos 2x}{1+\cos 2x} \Rightarrow f'\left(\frac{\pi}{3}\right) = ?$

- A) 3 B) $4\sqrt{3}$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{\sqrt{3}}{3}$ E) 16

3. $f(x) = \frac{1}{(\cos 2x - \sin 2x)^2} \Rightarrow f'\left(\frac{\pi}{24}\right) = ?$

- A) $4\sqrt{2}$ B) $4\sqrt{3}$ C) 12 D) 16 E) $8\sqrt{3}$

4. $f(x) = \left[\frac{2\sin^2 x - \tan 3x}{(1+\cos 2x)\sqrt{2+\sec^2 x}} \right]^{-6}$ olduğuna göre

$f'\left(\frac{\pi}{4}\right)$ kaçtır?

- A) 2 B) $\frac{5}{2}$ C) 3 D) $\frac{7}{2}$ E) 4

5. $x = \frac{\pi}{3}$

$f(x) = \sec x \Rightarrow \frac{d^2 f(x)}{dx^2} = ?$

- A) 8 B) 10 C) 12 D) 14 E) 16

6. $f(x) = \arctan\left(\frac{4\sin x}{3+5\cos x}\right) \Rightarrow f'\left(\frac{\pi}{3}\right) = ?$

- A) $\frac{4}{5}$ B) $\frac{6}{11}$ C) $\frac{8}{13}$ D) $\frac{8}{25}$ E) $\frac{16}{25}$

7. $f(x) = \arctan\left(\frac{1}{x}\right) \Rightarrow f'(2) = ?$

- A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{5}$ D) $-\frac{1}{5}$ E) $-\frac{1}{6}$

8. $f(x) = e^x \cdot \ln x \Rightarrow f'(x) = ?$

- A) $e^x \left(\ln x + \frac{1}{x}\right)$ B) $e^x \left(1 + \frac{1}{x}\right)$ C) $\ln x(e^x + 1)$
D) $\frac{e^x}{x}$ E) $e^x + \ln x$

9. $e^{x+y} = y^x \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{y}{x(x+y)}$ B) $\frac{y^2}{x(y-x)}$ C) $\frac{y^3}{x(y-x)}$
D) $\frac{y^3}{x^2-xy}$ E) $\frac{y^2}{x^2+y}$

10. $f(x) = \frac{x^3+x+2}{x^2+4}$

$\Rightarrow \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = ?$

- A) 2 B) 1 C) $\frac{5}{6}$ D) $\frac{13}{16}$ E) $\frac{7}{8}$

11. $f(x) = \frac{x^3 + 8}{x^2 + 2}$

$\Rightarrow \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = ?$

- A) $\frac{2}{9}$ B) $\frac{3}{5}$ C) $\frac{5}{6}$ D) 1 E) $\frac{3}{2}$

12. $f(x) = x^3 - 4|x| + 2x \Rightarrow f'(2) = ?$

- A) 8 B) 10 C) 12 D) 15 E) 24

13. $f(x) = x^3 - 6x + g(4x - 7)$

$g'(5) = -3$

$\Rightarrow f'(3) = ?$

- A) -6 B) -3 C) 0 D) 6 E) 9

14. $f(x) = x^2 + 1$

$g(x) = x^3 + 2x$

$\Rightarrow (f \circ g)'(1) = ?$

- A) 30 B) 28 C) 24 D) 18 E) 16

15. $f(x) = x^3 - x^2 - 12x + 7$

$\Rightarrow (f^{-1})'(7) = ?$

- A) $\frac{3}{16}$ B) $\frac{1}{86}$ C) $\frac{1}{28}$ D) $\frac{1}{24}$ E) $\frac{5}{96}$

16. $\left. \begin{array}{l} x = 6t - 4 \\ y = t^3 + 8 \end{array} \right\} \Rightarrow f'(8) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

17. $f(x) = x^5 - 2x^4 + x^3 - 2x^2 + 4x - 4$

$\Rightarrow (f^{-1})'(4) = ?$

- A) $\frac{1}{6}$ B) $\frac{1}{8}$ C) $\frac{1}{12}$ D) $\frac{1}{16}$ E) $\frac{1}{24}$

18. $f(x) = x \cdot \sqrt{x^3 + 8} \Rightarrow f'(2) = ?$

- A) $\frac{4}{9}$ B) $\frac{5}{2}$ C) 5 D) $\frac{13}{2}$ E) 7

19. $f(x) = \arctan(\sin x)$

$\cos a = \frac{2}{5} \Rightarrow f'(a) = ?$

- A) $\frac{7}{24}$ B) $\frac{5}{23}$ C) $\frac{3}{20}$ D) $\frac{2}{9}$ E) $\frac{1}{25}$

Yanıtlar / Answers

1. C	2. A	3. E	4. C	5. D	6. C
7. D	8. A	9. B	10. E	11. A	12. B
13. E	14. A	15. C	16. E	17. E	18. E
19. B					

1. $f(x) = \arcsin(\tan x)$

$\tan \theta = \frac{-1}{3}$

$\Rightarrow f'(\theta) = ?$

- A) $\frac{5\sqrt{2}}{6}$ B) $\frac{5\sqrt{3}}{3}$ C) $\frac{5}{6}$ D) $\frac{3}{16}$ E) $\frac{4}{9}$

2. $f(x) = e^{\sin(\ln x)} \Rightarrow f'(1) = ?$

- A) 1 B) 2 C) $\frac{1}{e}$ D) e E) π

3. $f(x) = x^{\sin x} \Rightarrow f'\left(\frac{\pi}{2}\right) = ?$

- A) $-\frac{\pi}{4}$ B) $-\frac{\pi}{3}$ C) $-\frac{\pi}{2}$ D) 1 E) $\frac{3\pi}{2}$

4. $y = e^x \cdot e^{\ln x} \Rightarrow \frac{dy}{dx} = ?$

- A) $e^x(x+1)$ B) $\frac{e^x}{x}$ C) $\frac{e^x \ln x}{x}$
D) $e^x(x+1)$ E) $e^x\left(1+\frac{1}{x}\right)$

5. $f(x) = (e^x)^{e^x} \Rightarrow f'(\ln 2) = ?$

- A) 24 B) 12 C) $4 + \ln 64$
D) $8 + \ln 256$ E) $64 \ln 2$

6. $y^x = e^{x+y} \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{y}{y-x}$ B) $\frac{y^2}{x(y-x)}$ C) $\frac{y^2}{y(y-x)}$
D) $\frac{y^2}{y(x-y)}$ E) $\frac{2y}{xy(1-x)}$

7. $xy = (x+y)^2 \Rightarrow \frac{dy}{dx} = ?$

- A) $-\frac{2x+y}{x+2y}$ B) $\frac{x-y}{x+y}$ C) $\frac{y-2x}{2y-x}$
D) $\frac{2xy}{x+y}$ E) $\frac{x+y}{y-2x}$

8. $f(x) = \frac{2}{x^2+1} \Rightarrow \lim_{h \rightarrow 0} \frac{f(-4+h) - f(-4)}{h} = ?$

- A) $\frac{4}{17}$ B) $\frac{6}{54}$ C) $\frac{12}{108}$ D) $\frac{16}{225}$ E) $\frac{16}{289}$

9. $f(x) = \frac{1}{x-1} \ (x \neq 1) \Rightarrow \lim_{t \rightarrow 0} \frac{f(t-2) - f(-2)}{t} = ?$

- A) $-\frac{1}{3}$ B) $-\frac{1}{6}$ C) $-\frac{1}{9}$ D) $\frac{1}{3}$ E) $\frac{1}{9}$

10. $\left. \begin{matrix} x = t - t^3 \\ y = 1 - t \end{matrix} \right\} \Rightarrow \frac{dy}{dx} \Big|_{t=1} = ?$

- A) $-\frac{2}{5}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 1

11. $\left. \begin{matrix} x = t^3 - 4t \\ y = t^3 \end{matrix} \right\} \Rightarrow \frac{d^2y}{dx^2} \Big|_{t=1} = ?$

- A) 6 B) 9 C) 12 D) 18 E) 24

12. $\left. \begin{matrix} x = \cos t \\ y = \sin t \end{matrix} \right\} \Rightarrow \frac{d^2y}{dx^2} = ?$

- A) $\tan t \sec(t)$ B) $\cot(t)$ C) $-\sec^3 t$
D) $\tan^2 t$ E) $4 - \operatorname{cosec}^3 t$

13. $y = \log(2x+1) \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{2}{2x+1}$ B) $\frac{2}{2x+1} \cdot \ln 10$ C) $\frac{2 \log e}{2x+1}$
D) $\frac{2}{(2x+1)^2}$ E) $\frac{\log e}{2x+1}$

14. $y = x^{\ln 3} \Rightarrow \frac{dy}{dx} = ?$

- A) $\ln 3 \cdot x^{\ln 3}$ B) $\frac{\ln 3}{x}$ C) $\frac{\ln^2 3}{x}$
D) $\frac{\ln^3 e}{x}$ E) $\frac{1}{x^{\ln 3}}$

15. $y = \ln x^3 + \ln^3 x \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{3(1+\ln x)}{x}$ B) $\frac{3(1+\ln^2 x)}{x}$ C) $\frac{1+\ln^2 x}{x^2}$
D) $\frac{3 \ln x}{x}$ E) $\frac{3(x+\ln x)}{x}$

16. $y = \ln[(x^2+2)^2 \cdot (x^3+x-1)] \Rightarrow \frac{dy}{dx} \Big|_{x=2} = ?$

- A) $\frac{37}{9}$ B) $\frac{8}{3}$ C) $\frac{25}{9}$ D) 3 E) 4

17. $y = \ln^2(2x+3) \Rightarrow \frac{dy}{dx} \Big|_{x=3} = ?$

- A) $\frac{\ln 3}{9}$ B) $\frac{4 \ln^3}{9}$ C) $\frac{8}{9}$ D) $\frac{16}{27}$ E) $\frac{8 \ln 3}{9}$

18. $y = \log_3(3x+1) \Rightarrow \frac{dy}{dx} = ?$

- A) $\frac{3 \log_3 e}{3x+1}$ B) $\frac{3}{3x+1}$ C) $\frac{3 \ln 3}{3x+1}$
D) $\frac{\log_3 9}{3x+1}$ E) $\frac{27}{3x+1}$

Yanıtlar / Answers

1. A	2. A	3. D	4. A	5. D	6. B
7. A	8. E	9. C	10. E	11. E	12. E
13. C	14. A	15. B	16. C	17. E	18. A

BELİRSİZ İNTEGRAL:

$\int f(x) dx = F(x) \Rightarrow F'(x) = f(x)$ 'tir. Her sabitin türevi sıfır olduğundan $F'(x) = f(x)$ ise $\int f(x) dx = F(x) + C$ ($C \in \mathbb{R}$)

olur.

$\int f(x) dx$ integralini hesaplamak, türevi $f(x)$ olan fonksiyonu bulmak demektir.

THE INDEFINITE INTEGRAL:

$\int f(x) dx = F(x) \Rightarrow F'(x) = f(x)$. Since the derivative of each constant equals zero, if $F'(x) = f(x)$ then we can state the following equality.

$\int f(x) dx = F(x) + C$. ($C \in \mathbb{R}$) calculating the integral

$\int f(x) dx$ means finding the function, the derivative of which is $f(x)$.

$k \in \mathbb{R} \Rightarrow$

$\int k \cdot f(x) dx = k \int f(x) dx$

$\int [f(x) \mp g(x)] dx = \int f(x) dx \mp \int g(x) dx$



BELİRSİZ İNTEGRALIN BAZI ÖZELLİKLER
SOME PROPERTIES of INDEFINITE INTEGRAL

1. $\int a dx = a \int dx = a \cdot x + C$ ($C \in \mathbb{R}$)

2. $\int a \cdot x^n dx = a \int x^n dx = \frac{a}{n+1} x^{n+1} + C$ ($C \in \mathbb{R}$)

3. $\int \frac{1}{x} dx = \ln|x| + C$, ($C \in \mathbb{R}$)

4. $\int e^x dx = e^x + C$, ($C \in \mathbb{R}$)

5. $\int a^x dx = \frac{a^x}{\ln a} + C = a^x \log_a e + C$, ($C \in \mathbb{R}$)

6. $\int \sin x dx = -\cos x + C$, ($C \in \mathbb{R}$)

7. $\int \cos x dx = \sin x + C$, ($C \in \mathbb{R}$)

8. $\int \frac{1}{\sin^2 x} dx = \int \operatorname{cosec}^2 x dx = -\cot x + C$, ($C \in \mathbb{R}$)

9. $\int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$, ($C \in \mathbb{R}$)

10. $\int \frac{1}{1+x^2} dx = \arctan x + C$, ($C \in \mathbb{R}$)

11. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$, ($C \in \mathbb{R}$)

12. $\int \frac{1}{\sqrt{1+x^2}} dx = \ln(x + \sqrt{1+x^2}) + C$, ($C \in \mathbb{R}$)

Örnek / Example:

$$\begin{aligned} & \int \left(x^2 - 3\sqrt{x} + \cos x + \frac{5}{x+1} \right) dx \\ &= \int x^2 dx - 3 \int \sqrt{x} dx + \int \cos x dx + 5 \int \frac{dx}{x+1} \\ &= \frac{x^3}{3} - 3 \int x^{\frac{1}{2}} dx + \sin x + 5 \ln |x+1| + C \\ &= \frac{x^3}{3} - 2x^{\frac{3}{2}} + \sin x + 5 \ln |x+1| + C \end{aligned}$$

Örnek / Example:

$$\begin{aligned} & \int \left(x^3 + \frac{1}{1+x^2} - \frac{1}{x+2} + \sin x \right) dx \\ &= \int x^3 dx + \int \frac{dx}{1+x^2} - \int \frac{dx}{x+2} + \int \sin x dx \\ &= \frac{x^4}{4} + \arctan x - \ln(x+2) - \cos x + C \end{aligned}$$



İNTEGRAL HESABININ TEMEL TEOREMLERİ THE BASIC THEOREMS of INTEGRAL CALCULUS

1. $\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$
2. $F(x) = \int_a^x f(t) dt, \quad x \in [a, b] \quad F'(x) = f(x)$
3. $F(x) = \int_a^{u(x)} f(t) dt \Rightarrow F'(x) = u'(x) \cdot f(u(x))$
4. $F(x) = \int_{v(x)}^{u(x)} f(t) dt \Rightarrow F'(x) = u'(x) \cdot f(u(x)) - v'(x) \cdot f(v(x))$

Örnek / Example:

$$\begin{aligned} & F(x) = \int_2^{3x} \cos(t^2) dt \Rightarrow F'(x) = 3 \cos(9x^2) \\ & F(x) = \int_1^{x^2} \frac{1}{1+\sqrt{1-t}} dt \Rightarrow F'(x) = \frac{2x}{1+\sqrt{1-x^2}} \\ & F(x) = \int_{\tan x}^0 \frac{1}{3+t} dt \Rightarrow F'(x) = \frac{-\sec^2 x}{3+\tan x} \end{aligned}$$



İNTEGRAL ALMA YÖNTEMLERİ METHODS of TAKING INTEGRAL

1. DEĞİŞKEN DEĞİŞTİRME YÖNTEMİ CHANGING the VARIABLE

$\int f(x) dx$ integralinde, $x = u(t)$ şeklinde bir değişken değıştirmesi yapıldığında, $x = u(t) \Rightarrow dx = u'(t) dt$ olur.

$$\int f(x) dx = \int f(u(t)) \cdot u'(t) \cdot dt \text{ olan integrali hesaplandıktan}$$

sonra sonuç tekrar x değışkeni cinsinden ifade edilir.

When $x = u(t)$ conversion $x = u(t)$ is applied in $\int f(x) dx$, dx becomes equal to $u'(t) \cdot dt$. Then the resulting integral is as the following:

$\int f(x) dx = \int f(u(t)) \cdot u'(t) \cdot dt$ after calculating this integral, we again convert the result to find the answer in terms of x .

Örnek / Example:

$$\int (x^2 + 2)^4 \cdot x dx = ?$$

Çözüm / Solution:

$$u = x^2 + 2 \Rightarrow du = 2x dx \Rightarrow x dx = \frac{du}{2}$$

$$\begin{aligned} \int (x^2 + 2)^4 \cdot x dx &= \int u^4 \cdot \frac{du}{2} \\ &= \frac{1}{2} \int u^4 du = \frac{1}{10} u^5 + C \\ &= \frac{1}{10} (x^2 + 2)^5 + C \end{aligned}$$

Örnek / Example:

$$\int \frac{x}{x^2 + 4} dx = ?$$

Çözüm / Solution:

$$u = x^2 + 4 \Rightarrow du = 2x dx \Rightarrow x dx = \frac{du}{2}$$

$$\int \frac{x}{x^2 + 4} dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln(x^2 + 4) + C$$

Örnek / Example:

$$\int \frac{dx}{9+x^2} = ?$$

Çözüm / Solution:

$$I = \int \frac{1}{9+x^2} dx = \frac{1}{9} \int \frac{1}{1+\left(\frac{x}{3}\right)^2} dx$$

$$\frac{x}{3} = t \Rightarrow dx = 3dt$$

$$I = \frac{1}{9} \int \frac{3}{1+t^2} dt = \frac{1}{3} \int \frac{1}{1+t^2} dt$$

$$I = \frac{1}{3} \arctan t + C = \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$$

2. KISMİ İNTEGRASYON YÖNTEMİ / INTEGRATION by PARTS

$$d(uv) = u dv + v du$$

$$uv = \int u dv + \int v du$$

$$\int u dv = uv - \int v du$$

Örnek / Example:

$$F(x) = \int \ln x dx \Rightarrow F(x) = ?$$

Çözüm / Solution:

$$u = \ln x \Rightarrow du = \frac{dx}{x}$$

$$dv = dx \Rightarrow v = x$$

$$\int u dv = uv - \int v du$$

$$= x \cdot \ln x - \int x \cdot \frac{dx}{x}$$

$$= x \cdot \ln x - x + C$$

$$= x \cdot (\ln x - 1) + C$$

Örnek / Example:

$$F(x) = \int \arctan x \cdot dx \Rightarrow F(x) = ?$$

Çözüm / Solution:

$$u = \arctan x \Rightarrow du = \frac{dx}{1+x^2}$$

$$d\theta = dx \Rightarrow \theta = x$$

$$F(x) = \int \arctan x dx = x \cdot \arctan x - \int \frac{x}{1+x^2} dx$$

$$1+x^2 = t \Rightarrow dt = 2x dx \Rightarrow x \cdot dx = \frac{1}{2} dt$$

$$F(x) = x \cdot \arctan x - \frac{1}{2} \int \frac{dt}{t}$$

$$F(x) = x \cdot \arctan x - \frac{1}{2} \ln |t| + C$$

$$F(x) = x \cdot \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

$$F(x) = x \cdot \arctan x - \ln \sqrt{1+x^2} + C$$

3. BASİT KESİRLERE AYIRMA YÖNTEMİ ile İNTEGRAL ALMAK

INTEGRATION by the METHOD of PARTIAL FRACTIONS

Örnek / Example:

$$\int \frac{5x-3}{x^2-2x-3} dx = ?$$

Çözüm / Solution:

$$\frac{5x-3}{x^2-2x-3} = \frac{5x-3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$
$$= \frac{(A+B)x+B-3A}{(x+1)(x-3)}$$

$$(A+B)x+B-3A=5x-3$$

$$\left. \begin{array}{l} A+B=5 \\ B-3A=-3 \end{array} \right\} \Rightarrow 4A=8 \Rightarrow A=2$$

$$A=2 \Rightarrow B=3$$

$$\int \frac{5x-3}{x^2-2x-3} dx = 2 \int \frac{dx}{x+1} + 3 \int \frac{dx}{x-3}$$
$$= 2 \ln |x+1| + 3 \ln |x-3| + C$$

Örnek / Example:

$$\int \frac{x}{x^3 - x^2 + x - 1} dx = ?$$

Çözüm / Solution:

$$\frac{x}{x^3 - x^2 + x - 1} = \frac{x}{(x^2 + 1)(x - 1)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x - 1}$$

$$= \frac{Ax^2 - Ax + Bx - B + Cx^2 + C}{(x^2 + 1)(x - 1)}$$

$$x = (A + C)x^2 + (B - A)x + C - B$$

$$\begin{cases} A + C = 0 \\ B - A = 1 \\ C - B = 0 \end{cases} \Rightarrow \begin{cases} A + C = 0 \\ C - A = 1 \end{cases} \Rightarrow C = \frac{1}{2}$$

$$C = \frac{1}{2}, A = -\frac{1}{2}, B = \frac{1}{2}$$

$$\int \frac{x}{x^3 - x^2 + x - 1} dx = \frac{1}{2} \int \frac{-x + 1}{x^2 + 1} dx + \frac{1}{2} \int \frac{1}{x - 1} dx$$

$$= \frac{-1}{2} \int \frac{x}{x^2 + 1} dx + \frac{1}{2} \int \frac{1}{x^2 + 1} dx + \frac{1}{2} \ln |x - 1| + C$$

$$x^2 + 1 = t \Rightarrow \frac{dt}{2} = x dx$$

$$\int \frac{x dx}{x^3 - x^2 + x - 1} = \frac{-1}{4} \int \frac{dt}{t} + \frac{1}{2} \arctan x + \frac{1}{2} \ln |x - 1| + C$$

$$\int \frac{x}{x^3 - x^2 + x - 1} dx = I$$

$$I = -\frac{1}{4} \ln(x^2 + 1) + \frac{1}{2} \arctan x + \frac{1}{2} \ln |x - 1| + C$$



BELİRLİ İNTEGRAL / THE DEFINITE INTEGRAL

Belirli integraller aşağıdaki gibi ifade edilir.

In general, integrals of the following type is called improper integral.

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$



BELİRLİ İNTEGRALIN ÖZELLİKLER

PROPERTIES of the IMPROPER INTEGRALS

$$1. \int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx, k \in \mathbb{R}$$

$$2. \int_a^b [f(x) \mp g(x)] dx = \int_a^b f(x) dx \mp \int_a^b g(x) dx$$

$$3. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, c \in (a, b)$$

Örnek / Example:

$$\int_0^2 \left(\frac{2x^3 + 8x + 1}{x^2 + 4} \right) dx = ?$$

Çözüm / Solution:

$$\int_0^2 \left(\frac{2x^3 + 8x + 1}{x^2 + 4} \right) dx = \int_0^2 \left(2x + \frac{1}{x^2 + 4} \right) dx$$

$$= 2 \int_0^2 x dx + \frac{1}{4} \int_0^2 \frac{dx}{1 + \left(\frac{x}{2} \right)^2}$$

$$2t = x \Rightarrow 2dt = dx$$

$$= x^2 \Big|_0^2 + \frac{1}{2} \int_0^2 \frac{dt}{1 + t^2}$$

$$= x^2 \Big|_0^2 + \frac{1}{2} \arctan \left(\frac{x}{2} \right) \Big|_0^2$$

$$= 4 + \frac{1}{2} (\arctan 1 - \arctan 0)$$

$$= 4 + \frac{1}{2} \cdot \frac{\pi}{4} = \frac{32 + \pi}{8}$$

Örnek / Example:

$$\int_{-3}^3 |x^2 - 4| dx = ?$$

Çözüm / Solution:

$$\begin{aligned} & \begin{array}{c} x \quad | \quad -3 \quad -2 \quad 2 \quad 3 \\ \quad \quad + \quad + \quad + \quad - \quad - \quad + \quad + \quad + \end{array} \\ & \int_{-3}^3 |x^2 - 4| dx = \int_{-3}^{-2} (x^2 - 4) dx + \int_{-2}^2 (4 - x^2) dx + \int_2^3 (x^2 - 4) dx \\ & = \left(\frac{x^3}{3} - 4x \right) \Big|_{-3}^{-2} + \left(4x - \frac{x^3}{3} \right) \Big|_{-2}^2 + \left(\frac{x^3}{3} - 4x \right) \Big|_2^3 \\ & = \left[\left(-\frac{8}{3} + 8 \right) - (-9 + 12) \right] + \left[\left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) \right] \\ & \quad + \left[(9 - 12) - \left(\frac{8}{3} - 8 \right) \right] \\ & = \frac{16}{3} - 3 + \frac{16}{3} + \frac{16}{3} - 3 + \frac{16}{3} - 3 = \frac{64}{3} - 6 = \frac{46}{3} \end{aligned}$$

Örnek / Example:

$$\int_0^{\sqrt{3}} \frac{1}{1+x^2} dx = ?$$

Çözüm / Solution:

$$\begin{aligned} \int_0^{\sqrt{3}} \frac{dx}{1+x^2} &= \arctan x \Big|_0^{\sqrt{3}} \\ &= \arctan \sqrt{3} - \arctan 0 = \frac{\pi}{3} \end{aligned}$$

Örnek / Example:

$$\int_2^4 |x-3| dx = ?$$

Çözüm / Solution:

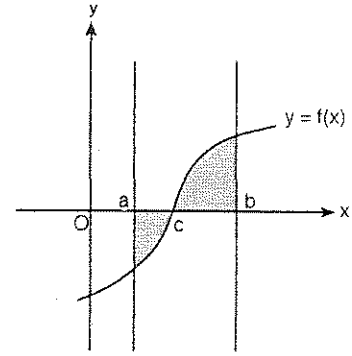
$$\begin{aligned} \int_2^4 |x-3| dx &= \int_2^3 (3-x) dx + \int_3^4 (x-3) dx \\ &= \left(3x - \frac{x^2}{2} \right) \Big|_2^3 + \left(\frac{x^2}{2} - 3x \right) \Big|_3^4 \\ &= \left[\left(9 - \frac{9}{2} \right) - (6 - 2) \right] + \left[(8 - 12) - \left(\frac{9}{2} - 9 \right) \right] = 1 \end{aligned}$$



BELİRLİ İNTEGRALIN UYGULAMALARI APPLICATIONS of the DEFINITE INTEGRAL

ALAN HESABI / AREA CALCULUS

$$1. \quad A = \int_a^b f(x) dx$$



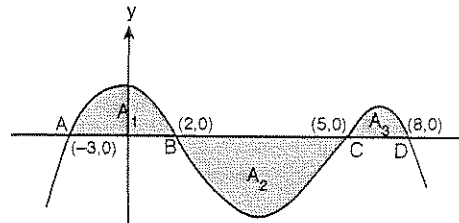
i. $[c, b]$ aralığında $f(x) \geq 0$

$$\text{In the interval } [c, b] \quad f(x) \geq 0 \Rightarrow A = \int_c^b f(x) dx$$

ii. $[a, c]$ aralığında $f(x) \leq 0$

$$\text{In the interval } [a, c] \quad f(x) \leq 0 \Rightarrow A = -\int_a^c f(x) dx$$

Örnek / Example:



$$A_1 = 12 \text{ birim}^2 / \text{unit}^2$$

$$A_2 = 18 \text{ birim}^2 / \text{unit}^2$$

$$A_3 = 9 \text{ birim}^2 / \text{unit}^2$$

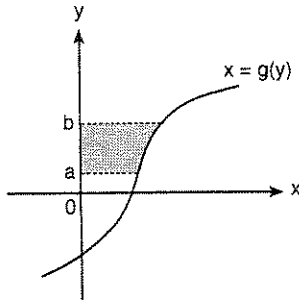
$$\Rightarrow \int_{-3}^8 f(x) dx = ?$$

Çözüm / Solution:

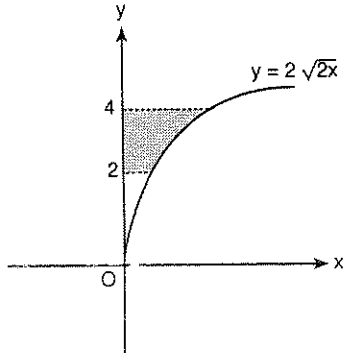
$$\int_{-3}^8 f(x) dx = \int_{-3}^2 f(x) dx + \int_2^5 f(x) dx + \int_5^8 f(x) dx$$

$$= 12 - 18 + 9 = 3 \text{ birim}^2 / \text{unit}^2$$

2. $A = \int_a^b |g(y)| dy$



Örnek / Example:



$\Rightarrow$ Taralı alan / The shaded area = ? $br^2(u^2)$

Çözüm / Solution:

$$y = 2\sqrt{2x} \Rightarrow y^2 = 8x$$

$$x = \frac{y^2}{8}$$

$$A = \int_2^4 |f(y)| dy$$

$$A = \int_2^4 \left| \frac{y^2}{8} \right| dy = \frac{y^3}{24} \Big|_2^4 = \frac{64}{24} - \frac{8}{24}$$

$$A = \frac{56}{24} = \frac{7}{3} br^2$$

ÇÖZÜMLÜ TEST / TEST WITH SOLUTIONS

1. $\int_{-1}^2 x^3 dx = ?$

A) $\frac{7}{2}$

B) $\frac{15}{4}$

C) 4

D) $\frac{17}{4}$

E) $\frac{9}{2}$

Çözüm / Solution:

$$\int_{-1}^2 x^3 dx = \frac{x^{3+1}}{3+1} \Big|_{-1}^2$$

$$= \frac{x^4}{4} \Big|_{-1}^2$$

$$= \frac{2^4}{4} - \frac{(-1)^4}{4}$$

$$= \frac{16}{4} - \frac{1}{4}$$

$$= \frac{15}{4}$$

Yanıt / Answer B

2. $\int \frac{x dx}{\sqrt[3]{x}} = ?$

A) $\frac{5}{3} \cdot x^{\frac{3}{5}} + C$

B) $15 \cdot x^{\frac{5}{3}} + C$

C) $\frac{3}{5} \cdot x^{\frac{5}{3}} + C$

D) $15 \cdot x^{\frac{5}{3}} + C$

E) $\frac{1}{3} \cdot x^{\frac{5}{3}} + C$

Çözüm / Solution:

$$\int \frac{x dx}{\sqrt[3]{x}} = \int \frac{x dx}{x^{\frac{1}{3}}}$$

$$= \int x^{1-\frac{1}{3}} dx$$

$$= \int x^{\frac{2}{3}} dx$$

$$= \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1}$$

$$= \frac{3}{5} x^{\frac{5}{3}} + C$$

Yanıt / Answer C

3. $\int (x^2 + 1) 2x dx = ?$

A) $3 \cdot (x^2 + 1)^3 + C$

B) $4 \cdot (x^2 + 1)^4 + C$

C) $\frac{(x^2 + 1)^3}{4} + C$

D) $\frac{(x^2 + 1)^4}{4} + C$

E) $\frac{(x^2 + 1)^4}{3} + C$

Çözüm / Solution:

$$\int (x^2 + 1)^3 2x dx = \int u^3 \cdot du$$

$$\left. \begin{array}{l} x^2 + 1 = u \\ 2x dx = du \end{array} \right\} \Rightarrow \frac{u^4}{4} + C$$

$$= \frac{(x^2 + 1)^4}{4} + C$$

Yanıt / Answer D

4. $\int \frac{dx}{x-3} = ?$

A) $\frac{1}{3} \cdot \ln|x-3| + C$

B) $\frac{1}{3} \cdot \ln|x+3| + C$

C) $3 \cdot \ln|x+3| + C$

D) $\ln|x+3| + C$

E) $\ln|x-3| + C$

Çözüm / Solution:

$$\int \frac{dx}{x-3} = \int \frac{du}{u}$$

$$= \ln|u| + C \quad (x-3=u, dx=du)$$

$$= \ln|x-3| + C$$

Yanıt / Answer E

5. $\int \frac{x+2}{x+1} dx = ?$

A) $x + \ln|x+1| + C$

B) $x - \ln|x+1| + C$

C) $2x + \ln|x+1| + C$

D) $x + 2 \cdot \ln|x+1| + C$

E) $x - 2 \cdot \ln|x+1| + C$

Çözüm / Solution:

$$\int \frac{x+2}{x+1} dx = \int \left(1 + \frac{1}{x+1} \right) dx$$

$$= x + \ln|x+1| + C$$

Yanıt / Answer A

6. $\int \sin 3x dx = ?$

A) $-\frac{1}{3} \cos 3x + C$

B) $\frac{1}{3} \cos 3x + C$

C) $-3 \cdot \cos 3x + C$

D) $3 \cos 3x + C$

E) $-\sin^3 3x + C$

Çözüm / Solution:

$$\int \sin 3x dx = \int \sin u \frac{du}{3}$$

$$\left. \begin{array}{l} 3x = u \\ 3dx = du \end{array} \right\} \Rightarrow \frac{1}{3} \int \sin u du$$

$$\Rightarrow -\frac{1}{3} \cdot (-\cos u) + C$$

$$\left. \begin{array}{l} dx = \frac{du}{3} \end{array} \right\} \Rightarrow -\frac{1}{3} \cos(3x) + C$$

Yanıt / Answer A

7. $\int \tan x dx = ?$

A) $\ln|\cos x| + C$

B) $-\ln|\cos x| + C$

C) $-\ln|\sin x| + C$

D) $\ln|\sin x| + C$

E) $\ln|\tan x| + C$

Çözüm / Solution:

$$\begin{aligned}\int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx \\ &= \int \frac{-du}{u} \\ \left. \begin{array}{l} \cos x = u \\ -\sin x \, dx = du \end{array} \right\} &= -\int \frac{du}{u} \\ &= -\ln|u| + c \\ &= -\ln|\cos x| + c\end{aligned}$$

Yanıt / Answer B

8. $\int_0^4 \sqrt{2x+1} \, dx = ?$

- A) $\frac{25}{3}$ B) $\frac{26}{3}$ C) 9 D) $\frac{28}{3}$ E) $\frac{29}{4}$

Çözüm / Solution:

$$\begin{aligned}\int_0^4 \sqrt{2x+1} \, dx &= \int \sqrt{u} \frac{du}{2} \\ &= \frac{1}{2} \int u^{\frac{1}{2}} \, du \\ \left. \begin{array}{l} 2x+1 = u \\ 2dx = du \\ dx = \frac{du}{2} \end{array} \right\} &\Rightarrow = \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \\ &= \frac{1}{3} \cdot u^{\frac{3}{2}} \\ &= \frac{1}{3} \sqrt{(2x+1)^3} \Big|_0^4 \\ &= \frac{1}{3} \left(\sqrt{(2 \cdot 4 + 1)^3} - \sqrt{(2 \cdot 0 + 1)^3} \right) \\ &= \frac{1}{3} \left(\sqrt{9^3} - \sqrt{1^3} \right) \\ &= \frac{1}{3} (27 - 1) \\ &= \frac{26}{3}\end{aligned}$$

Yanıt / Answer B

9. $\int_0^2 |x^2 - 1| \, dx = ?$

- A) -4 B) -2 C) 2 D) 4 E) 6

Çözüm / Solution:

$$\begin{aligned}\int_0^2 |x^2 - 1| \, dx &= \int_0^1 (-x^2 + 1) \, dx + \int_1^2 (x^2 - 1) \, dx \\ &= \left(-\frac{x^3}{3} + x \right) \Big|_0^1 + \left(\frac{x^3}{3} - x \right) \Big|_1^2 \\ &= \left(-\frac{1}{3} + 1 \right) + \left[\left(\frac{8}{3} - 2 \right) - \left(\frac{1}{3} - 1 \right) \right] \\ &= \frac{2}{3} + \left[\frac{2}{3} + \frac{2}{3} \right] \\ &= \frac{2}{3} + \frac{4}{3} = 2\end{aligned}$$

Yanıt / Answer C

10. $\int \frac{x}{x^2 + 1} \, dx = ?$

- A) 0 B) $\ln \sqrt{\frac{1}{2}}$ C) $\ln \sqrt{2}$
D) $\ln \sqrt{3}$ E) $\ln 2$

Çözüm / Solution:

$$\begin{aligned}\int_0^1 \frac{x}{x^2 + 1} \, dx &= \int \frac{\frac{du}{2}}{u} \\ &= \frac{1}{2} \int \frac{du}{u} \\ \left. \begin{array}{l} x^2 + 1 = u \\ 2xdx = du \\ xdx = \frac{du}{2} \end{array} \right\} &\Rightarrow = \frac{1}{2} \cdot \ln|u| \Big|_0^1 \\ &= \frac{1}{2} \cdot \ln|x^2 + 1| \Big|_0^1 \\ &= \frac{1}{2} \cdot [\ln 2 - \ln 1] \\ &= \frac{1}{2} \cdot \ln 2 \\ &= \ln \sqrt{2}\end{aligned}$$

Yanıt / Answer C

11. $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)^3} dx = ?$

- A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) $\frac{1}{5}$

Çözüm / Solution:

$$\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)^3} dx = \int_0^{\pi/2} \frac{du}{u^3}$$

$$= \int_0^{\pi/2} \frac{1}{2} u^{-3} du \quad \begin{array}{l} 1 + \sin x = u \\ \cos x dx = du \end{array}$$

$$= \frac{u^{-2}}{-2} \Big|_0^{\pi/2}$$

$$= -\frac{1}{2u^2} \Big|_0^{\pi/2}$$

$$= -\frac{1}{2(1 + \sin x)^2} \Big|_0^{\pi/2}$$

$$= -\frac{1}{2} \left(\frac{1}{1 + \sin \frac{\pi}{2}} - \frac{1}{1 + \sin 0} \right)$$

$$= -\frac{1}{2} \left(\frac{1}{2} - 1 \right) = -\frac{1}{2} \cdot \left(-\frac{1}{2} \right) = \frac{1}{4}$$

Yanıt / Answer D

12. $\int_0^2 \frac{dx}{4 + x^2} = ?$

- A) $\frac{\pi}{4}$ B) $\frac{3\pi}{4}$ C) $\frac{\pi}{8}$ D) 2 E) $\frac{1}{8}$

Çözüm / Solution:

$$\int_0^2 \frac{dx}{4 + x^2} = \frac{1}{4} \Big|_0^2 \frac{dx}{1 + \left(\frac{x}{2}\right)^2}$$

$$= \frac{1}{4} \int_0^2 \frac{2du}{1+u} = \frac{1}{2} \arctan u \Big|_0^2$$

$$= \frac{1}{2} \arctan \frac{x}{2} \Big|_0^2$$

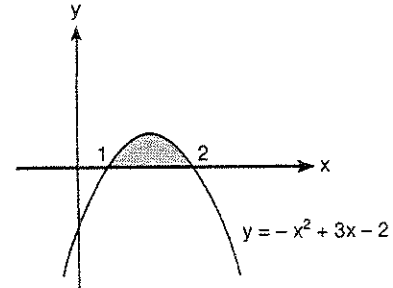
$$= \frac{1}{2} \arctan 1 - \frac{1}{2} \arctan 0$$

$$= \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2} \cdot 0$$

$$= \frac{\pi}{8}$$

Yanıt / Answer C

13.



Taralı alan / The shaded area = ?

- A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) $\frac{1}{5}$

Çözüm / Solution:

$$S = \int_1^2 (-x^2 + 3x - 2) dx$$

$$= \left(-\frac{x^3}{3} + 3 \cdot \frac{x^2}{2} - 2x \right) \Big|_1^2$$

$$= \left(-\frac{2^3}{3} + 3 \cdot \frac{2^2}{2} - 2 \cdot 2 \right) - \left(-\frac{1^3}{3} + 3 \cdot \frac{1^2}{2} - 2 \cdot 1 \right)$$

$$= \left(-\frac{8}{3} + 6 - 4 \right) - \left(-\frac{1}{3} + \frac{3}{2} - 2 \right)$$

$$= -\frac{8}{3} + 2 + \frac{1}{3} + \frac{1}{2}$$

$$= -\frac{7}{3} + \frac{5}{2}$$

$$= \frac{1}{6}$$

Yanıt / Answer D

14.

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1-x^2}} = ?$$

- A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{2\pi}{3}$ D) π E) $\frac{5\pi}{6}$

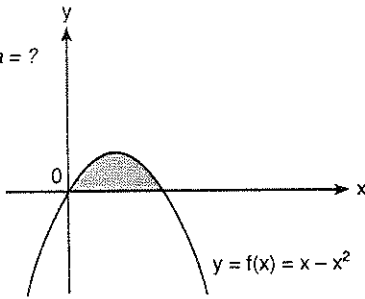
Çözüm / Solution:

$$\begin{aligned} \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1-x^2}} &= \left[\arcsin \frac{x}{1} \right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \\ &= \arcsin \frac{\sqrt{3}}{2} - \arcsin \frac{1}{2} \\ &= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \end{aligned}$$

Yanıt / Answer A

15.

Taralı alan = ?
The shaded area = ?



- A) 4 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{6}$

Çözüm / Solution:

$$\begin{aligned} x^2 &= x \\ x^2 - x &= 0 \\ x \cdot (x-1) &= 0 \\ x &= 0 \text{ veya (or) } x = 1 \end{aligned}$$

$$\begin{aligned} S &= \int_0^1 (x - x^2) dx \\ &= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

Yanıt / Answer E

$$16. \int_{-1}^1 (x^2 + 2x - 1) dx = ?$$

- A) 2 B) 1 C) $\frac{4}{3}$ D) $-\frac{1}{3}$ E) $-\frac{4}{3}$

Çözüm / Solution:

$$\begin{aligned} \int_{-1}^1 (x^2 + 2x - 1) dx &= \left[\frac{x^3}{3} + \frac{2 \cdot x^2}{2} - x \right]_{-1}^1 \\ &= \left(\frac{1}{3} + 1 - 1 \right) - \left(-\frac{1}{3} + 1 - 1 \right) \\ &= \frac{1}{3} + \frac{1}{3} - 2 \\ &= \frac{2}{3} - 2 \\ &= -\frac{4}{3} \end{aligned}$$

Yanıt / Answer E

$$17. \int x \cdot \sin x dx = ?$$

- A) $-x \cdot \cos x + \sin x + C$
B) $-x \cdot \sin x + \cos x + C$
C) $-x \cdot \cos x + \cos x + C$
D) $x \cdot \cos x + \sin x + C$
E) $-x \cdot \cos x - \sin x + C$

Çözüm / Solution:

$$\begin{aligned} x &= u, \sin x dx = dv \\ dx &= du, -\cos x = v \\ \int x \cdot \sin x dx &= x \cdot (-\cos x) - \int (-\cos x) dx \\ &= -x \cdot \cos x + \int \cos x dx \\ &= -x \cdot \cos x + \sin x + C \end{aligned}$$

Yanıt / Answer A

18. $\int_0^{\pi} \frac{dx}{x^2 - 4} = ?$

- A) $2 \cdot \ln \left| \frac{x+2}{x-2} \right| + C$ B) $\frac{1}{4} \cdot \ln \left| \frac{x+2}{x-2} \right| + C$
 C) $\frac{1}{4} \cdot \ln \left| \frac{x+2}{x-2} \right| + C$ D) $\frac{1}{2} \cdot \ln \left| \frac{x+2}{x-2} \right| + C$
 E) $\frac{1}{2} \cdot \ln \left| \frac{x-2}{x+2} \right| + C$

Çözüm / Solution:

$$\frac{1}{x^2 - 4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$\begin{cases} A+B=0 \\ 2A-2B=1 \end{cases} \Rightarrow A = -\frac{1}{4}, B = \frac{1}{4}$$

$$\begin{aligned} \int \frac{dx}{x^2 - 4} &= \int \left(\frac{\frac{1}{4}}{x-2} - \frac{\frac{1}{4}}{x+2} \right) dx \\ &= \frac{1}{4} \cdot \int \left(\frac{1}{x-2} - \frac{1}{x+2} \right) dx \\ &= \frac{1}{4} \cdot (\ln |x-2| - \ln |x+2|) \\ &= \frac{1}{4} \cdot \ln \left| \frac{x-2}{x+2} \right| + C \end{aligned}$$

Yanıt / Answer B

19. $f'(x) = x^2 + 3x$,

$f(1) = 5 \Rightarrow f(2) = ?$

- A) $\frac{23}{2}$ B) $\frac{35}{3}$ C) $\frac{71}{6}$ D) 12 E) $\frac{73}{6}$

Çözüm / Solution:

$$f'(x) = x^2 + 3x$$

$$\int f'(x) dx = \int (x^2 + 3x) dx$$

$$f(x) = \frac{x^3}{3} + \frac{3x^2}{2} + c$$

$$f(1) = \frac{1}{3} + \frac{3}{2} + c = 5$$

$$c = 5 - \frac{11}{6}$$

$$c = \frac{19}{6}$$

$$f(2) = \frac{2^3}{3} + \frac{3 \cdot 2^2}{2} + \frac{19}{6} = \frac{71}{6}$$

Yanıt / Answer C

20. $\int_a^b (4x-1) dx = 12, a-b=-4 \Rightarrow a \cdot b = ?$

- A) 4 B) 3 C) 0 D) -3 E) -4

Çözüm / Solution:

$$\int_a^b (4x-1) dx = 12$$

$$\left(\frac{4x^2}{2} - x \right) \Big|_a^b = 12$$

$$(2b^2 - b) - (2a^2 - a) = 12$$

$$2b^2 - b - 2a^2 + a = 12$$

$$2 \cdot (b^2 - a^2) + a - b = 12$$

$$2 \cdot (b-a) \cdot (b+a) + a - b = 12$$

$$2 \cdot 4 \cdot (b+a) - 4 = 12$$

$$8 \cdot (b+a) = 16$$

$$b+a = 2$$

$$+ \quad a-b = -4$$

$$2a = -2$$

$$a = -1, b = 3$$

$$a \cdot b = -1 \cdot 3 = -3$$

Yanıt / Answer D

21. $\int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x) dx = ?$

- A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) $\frac{1}{8}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

Çözüm / Solution:

$$\begin{aligned} \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x) dx &= \int_0^{\frac{\pi}{4}} \cos 2x dx \\ &= \frac{\sin 2x}{2} \Big|_0^{\frac{\pi}{4}} \\ &= \frac{1}{2} (\sin 2x) \Big|_0^{\frac{\pi}{4}} \\ &= \frac{1}{2} \left(\sin \frac{\pi}{2} - \sin 0 \right) \\ &= \frac{1}{2} (1 - 0) \\ &= \frac{1}{2} \end{aligned}$$

Yanıt / Answer E

22. $\frac{d}{dx} \left(\int_1^8 (x^3 - 5x^2) dx \right) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

Çözüm / Solution:

$$\int_1^8 (x^3 - 5x^2) dx = a, \quad a \in \mathbb{R} \Rightarrow \frac{d}{dx} a = 0$$

Yanıt / Answer C

YÖS SORULARI / YÖS QUESTIONS

1. $8 \int_0^{\frac{\pi}{12}} (\sin x \cdot \cos x \cdot \cos 2x) dx = ?$

- A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 0 D) 1 E) -1

(YÖS 1992)

Çözüm / Solution:

$$\sin x \cdot \cos x = \frac{\sin 2x}{2}$$

$$\frac{\sin 2x}{2} \cdot \cos 2x = \frac{1}{2} \cdot \sin 2x \cdot \cos 2x$$

$$= \frac{1}{2} \cdot \frac{\sin 4x}{2}$$

$$= \frac{1}{4} \cdot \sin 4x$$

$$8 \int (\sin x \cdot \cos x \cdot \cos 2x) dx = 8 \int \frac{1}{4} \cdot \sin 4x dx$$

$$= 2 \int \sin 4x dx$$

$$= 2 \left(-\frac{\cos 4x}{4} \right)$$

$$= -\frac{\cos 4x}{2} \Big|_0^{\frac{\pi}{12}}$$

$$= -\frac{1}{2} \left(\cos 4 \cdot \frac{\pi}{12} - \cos 4 \cdot 0 \right)$$

$$= -\frac{1}{2} \left(\cos \frac{\pi}{3} - \cos 0 \right)$$

$$= -\frac{1}{2} \left(\frac{1}{2} - 1 \right)$$

$$= \frac{1}{4}$$

Yanıt / Answer A

2. $\int_0^1 \frac{5x^2 dx}{\sqrt{1-x^6}} = ?$

- A) $\frac{3\pi}{2}$ B) $\frac{2\pi}{2}$ C) $\frac{4\pi}{3}$
D) $\frac{3\pi}{4}$ E) $\frac{5\pi}{6}$

(YÖS 1994)

Çözüm / Solution:

$$\begin{aligned} \int_0^1 \frac{5x^2 dx}{\sqrt{1-x^6}} &= \int \frac{5x^2 dx}{\sqrt{1-(x^3)^2}} \\ x^3 &= u \\ 3x^2 dx &= du \\ x^2 dx &= \frac{du}{3} \end{aligned} \Rightarrow \begin{aligned} &= \int_0^1 \frac{5 \cdot \frac{du}{3}}{\sqrt{1-u^2}} \\ &= \frac{5}{3} \int_0^1 \frac{du}{\sqrt{1-u^2}} \\ &= \frac{5}{3} \arcsin u \\ &= \frac{5}{3} \arcsin x^3 \Big|_0^1 \\ &= \frac{5}{3} (\arcsin 1 - \arcsin 0) \\ &= \frac{5}{3} \left(\frac{\pi}{2} - 0 \right) \\ &= \frac{5\pi}{6} \end{aligned}$$

Yanıt / Answer E

3. $\int \frac{x^2 \cdot \ln(x^3+1)}{x^3+1} dx = ?$

- A) $\frac{[\ln(x^3+1)]^2}{6} + C$ B) $\ln(x^3+1)^2 + C$
C) $\frac{1}{3} \ln(x^3+1) + C$ D) $\frac{1}{12} \ln(x^3+1) + C$
E) $\frac{1}{18} \ln(x^3+1) + C$

(YÖS 1995)

Çözüm / Solution:

$$\begin{aligned} \int \frac{x^2 \cdot \ln(x^3+1)}{x^3+1} dx &= \int \frac{\ln u}{u} \cdot \frac{du}{3} \\ &= \frac{1}{3} \int \frac{\ln u}{u} du \\ x^3+1 &= u \\ 3x^2 dx &= du \\ x^2 dx &= \frac{du}{3} \\ \ln u &= t \\ \frac{1}{u} du &= dt \end{aligned} \Rightarrow \begin{aligned} &= \frac{1}{3} \int t dt \\ &= \frac{1}{3} \cdot \frac{t^2}{2} \\ &= \frac{1}{6} t^2 \\ &= \frac{1}{6} (\ln u)^2 \\ &= \frac{1}{6} (\ln(x^3+1))^2 + C \end{aligned}$$

Yanıt / Answer A

4. $\int_1^4 \left(2x - \frac{1}{\sqrt{x}} \right) dx = ?$

- A) $\frac{13}{2}$ B) $\frac{7}{2}$ C) 9 D) 11 E) 13

(YÖS 1998)

Çözüm / Solution:

$$\begin{aligned} \int_1^4 \left(2x - \frac{1}{\sqrt{x}} \right) dx &= 2 \int_1^4 x dx - \int_1^4 x^{-\frac{1}{2}} dx \\ &= 2 \cdot \frac{x^2}{2} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \Big|_1^4 \\ &= \left(x^2 - 2\sqrt{x} \right) \Big|_1^4 \\ &= 4^2 - 2 \cdot \sqrt{4} - (1^2 - 2\sqrt{1}) \\ &= 16 - 4 + 1 \\ &= 13 \end{aligned}$$

Yanıt / Answer E

5. $\int \frac{e^x}{3+5e^x} dx = ?$

- A) $\ln(5-e^x) + C$ B) $\ln(3-e^x) + C$
C) $\ln(3+e^x) + C$ D) $\frac{1}{10} \ln(3+e^x) + C$
E) $\frac{1}{5} \ln(3+5e^x) + C$

(YÖS 2001)

Çözüm / Solution:

$$\left. \begin{array}{l} u = 3 + 5e^x \\ du = 5e^x dx \\ \frac{du}{5} = e^x dx \end{array} \right\} \Rightarrow \int \frac{e^x}{3+5e^x} dx = \frac{1}{5} \int \frac{du}{u}$$

$$= \frac{1}{5} \ln u + c = \frac{1}{5} \ln (3 + 5e^x) + c$$

Yanıt / Answer E

6. $\int \cos(3x-2) dx = ?$

A) $\frac{1}{3} \sin(3x-2) + C$ B) $\ln(3-e^x) + C$
 C) $\ln(3+e^x) + C$ D) $\frac{1}{0} \ln(3+e^x) + C$
 E) $\frac{1}{5} \ln(3+5e^x) + C$

(YÖS 2001)

Çözüm / Solution:

$$\left. \begin{array}{l} u = 3x - 2 \\ du = 3dx \\ \frac{du}{3} = dx \end{array} \right\} \Rightarrow \int \cos u \frac{du}{3} = \frac{1}{3} \sin u + C$$

$$= \frac{1}{3} \sin(3x-2) + C$$

Yanıt / Answer D

7. $\int_1^3 (|x-2|+2) dx = ?$

A) $\frac{7}{2}$ B) $\frac{9}{2}$ C) 4 D) 5 E) 7

(YÖS 2002)

Çözüm / Solution:

$$\int_1^3 (|x-2|+2) dx = \int_1^2 (-x+2+2) dx + \int_2^3 (x-2+2) dx$$

$$= \int_1^2 (-x+4) dx + \int_2^3 x dx = \left(\frac{-x^2}{2} + 4x \right) \Big|_1^2 + \left(\frac{x^2}{2} \right) \Big|_2^3$$

$$= \frac{9}{2} + \frac{5}{2} = 7$$

Yanıt / Answer E

8. $\int \frac{x dx}{x^2-1} = ?$

A) $\frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C$ B) $\frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C$
 C) $\frac{1}{2} \ln |x^2-1| + C$ D) $\frac{1}{2} \ln |x^2+1| + C$
 E) $\frac{3}{2} \ln |x^2-1| + C$

(YÖS 2003)

Çözüm / Solution:

$$\int \frac{x dx}{x^2-1} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln |u| + C$$

$$= \frac{1}{2} \ln |x^2-1| + C$$

Yanıt / Answer C

9. $\int_0^{\frac{\pi}{2}} \cos x \cdot \sin x \cdot e^{\sin^2 x} dx = ?$

A) $e-2$ B) $e+2$ C) $\frac{1}{4}(e-1)$
 D) $\frac{1}{2}(e+1)$ E) $\frac{1}{2}(e-1)$

(YÖS 2003)

Çözüm / Solution:

$$\sin^2 x = u \Rightarrow 2 \sin x \cos x dx = du$$

$$\sin x \cos x dx = \frac{du}{2}$$

$$\int_0^{\frac{\pi}{2}} \cos x \cdot \sin x \cdot e^{\sin^2 x} dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} e^u du = \frac{1}{2} \int_0^{\frac{\pi}{2}} e^u$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} e^u \sin^2 x$$

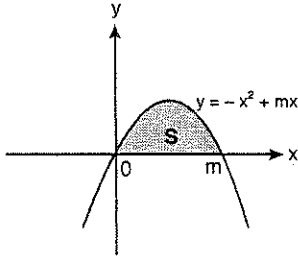
$$= \frac{1}{2} e^{(\sin(\frac{\pi}{2}))^2} - \frac{1}{2} e^{\sin^2 0}$$

$$= \frac{1}{2} e - \frac{1}{2}$$

$$= \frac{1}{2}(e-1)$$

Yanıt / Answer E

10.

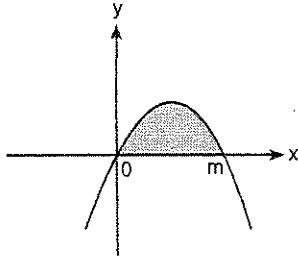


$$S = 288 \Rightarrow m = ?$$

- A) 10 B) 12 C) 13 D) 14 E) 15

(YÖS 2003)

Çözüm / Solution:



$$\begin{aligned} \int_0^m (-x^2 + mx) dx &= \left[-\frac{x^3}{3} + \frac{mx^2}{2} \right]_0^m \\ &= \left(-\frac{m^3}{3} + \frac{m^3}{2} \right) - (0+0) \\ &= 288 \end{aligned}$$

$$\frac{m^3}{6} = 288 \Rightarrow m = 12$$

Yanıt / Answer B

$$11. \int_{-2}^2 |x^2 - 4| dx = ?$$

- A) $\frac{16}{3}$ B) $\frac{29}{3}$ C) $\frac{32}{3}$
D) 8 E) 16

(YÖS 2004)

Çözüm / Solution:

$$\begin{aligned} \int_{-2}^2 |x^2 - 4| dx &= \int_{-2}^2 (x^2 - 4) dx = \left(\frac{x^3}{3} - 4x \right) \Big|_{-2}^2 \\ &= \left[\left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) \right] = \frac{32}{3} \end{aligned}$$

Yanıt / Answer C

$$12. \int \frac{x}{\sqrt{16-x^2}} dx = ?$$

- A) $\sqrt{4+x^2} + C$ B) $\sqrt{16+x^2} + C$
C) $-\sqrt{4-x^2} + C$ D) $-\sqrt{16-x^2} + C$
E) $-2\sqrt{16-x^2} + C$

(YÖS 2004)

Çözüm / Solution:

$$16 - x^2 = u$$

$$-2x dx = du$$

$$x dx = -\frac{du}{2}$$

$$\int x \frac{x}{\sqrt{16-x^2}} dx = \int \frac{-\frac{du}{2}}{\sqrt{u}}$$

$$= -\frac{1}{2} \int u^{-\frac{1}{2}} du = -\frac{1}{2} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -u^{\frac{1}{2}} + C = -(16-x^2)^{\frac{1}{2}} + C$$

$$= -\sqrt{16-x^2} + C$$

Yanıt / Answer D

$$13. \int x \cdot f(x) dx = 2x^3 + 4x^2 + C$$

$$f(x) = ?$$

- A) $6x + 4$ B) $6x + 8$ C) $6x + 13$
D) $\frac{2}{3}x^3 + 2x$ E) $\frac{2}{3}x^3 + 2x^2$

(YÖS 2005)

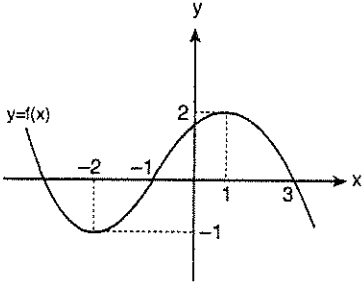
Çözüm / Solution:

$$\int x f(x) dx = 2x^3 + 4x^2 + C$$

$$\Rightarrow x f(x) = 6x^2 + 8x$$

$$f(x) = \frac{6x^2 + 8x}{x} = 6x + 8$$

Yanıt / Answer B

14. $\frac{df}{dx} = f'(x)$
 $\int_{-2}^1 f'(x) dx = ?$
- 
- A) 0 B) 1 C) 2 D) 3 E) 5

(YÖS 2005)

Çözüm / Solution::

$$\int_{-2}^1 f'(x) dx = f(x) \Big|_{-2}^1$$

$$= f(1) - f(-2) = 2 - (-1) = 3$$

Yanıt / Answer D

15. $\int_0^{\frac{\pi}{4}} e^{\cos 4x} \cdot \sin 4x dx = ?$

- A) $\frac{1-e^2}{4}$ B) $\frac{e^2-1}{4e}$ C) e
D) e^2 E) $e^2 - 1$

(YÖS 2005)

Çözüm / Solution:

$$\int_0^{\frac{\pi}{4}} e^{\cos 4x} \cdot \sin 4x dx$$

$$\cos 4x = u \Rightarrow -4 \sin 4x dx = du$$

$$\sin 4x \cdot dx = -\frac{du}{4}$$

$$\int_0^{\frac{\pi}{4}} e^u \left(-\frac{du}{4} \right) = -\frac{1}{4} e^u \Big|_{x=0}^{\frac{\pi}{4}}$$

$$= -\frac{1}{4} (e^{\cos 4 \cdot \frac{\pi}{4}} - e^{\cos 4 \cdot 0})$$

$$= -\frac{1}{4} (e^{-1} - e)$$

$$= -\frac{1}{4} \left(\frac{1}{e} - e \right) = \frac{e^2 - 1}{4e}$$

Yanıt / Answer B

16. $f(1) = 9$
 $f'(x) = 6x + 4$
 $f(0) = ?$
A) 0 B) 1 C) 2 D) 3 E) 4
(YÖS 2006)

Çözüm / Solution:

$$f'(x) = 6x + 4$$

$$\Rightarrow f(x) = \int 6x + 4$$

$$= 3x^2 + 4x + c$$

$$f(1) = 3 \cdot 1 + 4 \cdot 1 + c = 9$$

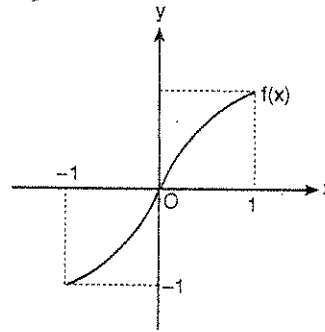
$$\Rightarrow c = 2$$

$$f(x) = 3x^2 + 4x + 2$$

$$f(0) = 3 \cdot 0 + 4 \cdot 0 + 2 = 2$$

Yanıt – Answer C

17.



$$\int_{-1}^1 [f(x)] dx = ?$$

- A) -2 B) -1 C) 0 D) 1 E) 2

(YÖS 2006)

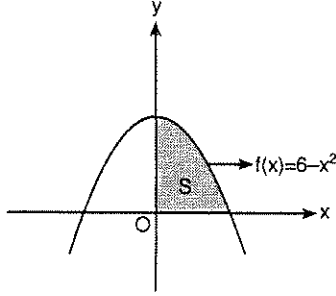
Çözüm / Solution:

$$\begin{aligned} \int_{-1}^1 [f(x)] dx &= \int_{-1}^0 -1 dx + \int_0^1 1 dx \\ &= -x \Big|_{-1}^0 = 1 \end{aligned}$$

Yanıt / Answer D

18. $f(x) = 6 - x^2$

$S = ? \text{ cm}^2$



- A) $2\sqrt{6}$ B) $4\sqrt{6}$ C) $6\sqrt{6}$ D) 6 E) 12

(YÖS 2006)

Çözüm / Solution:

$6 - x^2 = 0$

$x_1 = \sqrt{6}$,

$x_2 = -\sqrt{6}$

$$S = \int_{-\sqrt{6}}^{\sqrt{6}} (6 - x^2) dx$$

$$= 6x - \frac{x^3}{3} \Big|_{-\sqrt{6}}^{\sqrt{6}} = 4\sqrt{6}$$

Yanıt / Answer B

19. $f(x) = x$

$\int_1^t d(f^{-1}(x)) = 12$

$\Rightarrow t = ?$

- A) 9 B) 10 C) 11 D) 12 E) 13

(YÖS 2006)

Çözüm / Solution:

$f(x) = x \Rightarrow f^{-1}(x) = x$

$\int_1^t d(f^{-1}(x)) = \int_1^t dx = x \Big|_1^t = 12$

$\Rightarrow t - 1 = 12$

$\Rightarrow t = 13$

Yanıt / Answer E

20.

$\int_{\sqrt{2}}^{\sqrt{6}} x \sqrt{x^2 - 2} dx = ?$

A) $\frac{4}{3}$

B) $\frac{7}{3}$

C) $\frac{8}{3}$

D) $\frac{5}{4}$

E) $\frac{9}{4}$

(YÖS 2007)

Çözüm / Solution:

$\int_{\sqrt{2}}^{\sqrt{6}} x \sqrt{x^2 - 2} dx$

$x^2 - 2 = u$

$20 dx = du \Rightarrow x \cdot dx = \frac{du}{2}$

$\int_{\sqrt{2}}^{\sqrt{6}} \sqrt{u} \cdot \frac{du}{2} = \frac{1}{2} \int_{\sqrt{2}}^{\sqrt{6}} u^{\frac{1}{2}} du$

$= \frac{1}{2} \left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) \Big|_{\sqrt{2}}^{\sqrt{6}}$

$= \frac{1}{3} (x^2 - 2)^{\frac{3}{2}} \Big|_{\sqrt{2}}^{\sqrt{6}} = \frac{1}{3} (8 - 0) = \frac{8}{3}$

Yanıt / Answer C

21. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1 - \cos x}} dx = ?$

A) 1

B) 2

C) 3

D) $\sqrt{2}$

E) $2\sqrt{2}$

(YÖS 2007)

Çözüm / Solution:

$\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1 - \cos x}} dx$

$1 - \cos x = u$

$\sin x \cdot dx = du$

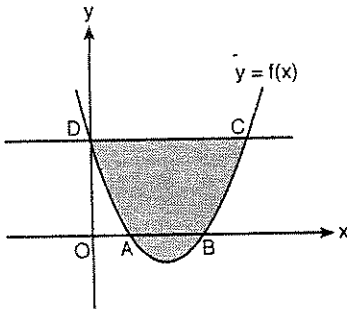
$\int_0^{\frac{\pi}{2}} \frac{du}{\sqrt{u}} = 2\sqrt{u} \Big|_0^{\frac{\pi}{2}} = 2\sqrt{(1 - \cos x)} \Big|_0^{\frac{\pi}{2}}$

$= 2(\sqrt{1 - 0} - \sqrt{1 - 1})$

$= 2$

Yanıt / Answer B

22.



DC // AB, $f(x) = x^2 - 5x + 6 \Rightarrow A(ABCD) = ? \text{ br}^2$

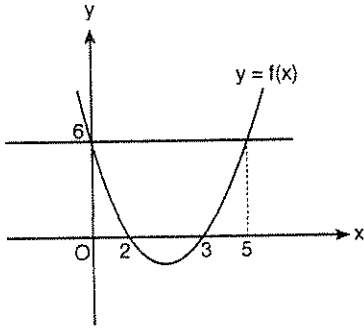
A) 16

B) 24

C) $\frac{52}{3}$ D) $\frac{62}{3}$ E) $\frac{82}{3}$

(YÖS 2007)

Çözüm / Solution:



$$A(ABCD) = 6 \cdot 5 - \int_0^5 (x^2 - 5x + 6) dx = \int_0^5 (6 - (x^2 - 5x + 6)) dx$$

$$= 30 - \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x \right) \Big|_0^5 = \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x \right) \Big|_3^5$$

$$= 30 - \frac{14}{3} - \frac{14}{3} = \frac{62}{3}$$

Yanıt / Answer D

1. $\int x f(x) dx = x^3 + 3x^2 + 4x + 6 \Rightarrow f(x) = ?$

Yanıt / Answer : $3x + \frac{4}{x} + 6$

2. $f'(x) = 4x^3 + 6x^2 + 2x + 3$ ve (and) $f(2) = 56 \Rightarrow f(x) = ?$

Yanıt / Answer : $x^4 + 2x^3 + x^2 + 3x + 14$

3. $\int \frac{x^2 + 2x + 1}{x^2 + 1} dx = ?$

Yanıt / Answer : $x + \ln(x^2 + 1) + C$

4. $\int \frac{x}{2x-1} dx = ?$

Yanıt / Answer : $\frac{1}{2}x + \frac{1}{4}\ln|2x-1| + C$

5. $\int \frac{(x+1)}{x^2 + 2x + 2} dx = ?$

Yanıt / Answer : $\frac{1}{2}\ln(x^2 + 2x + 2) + C$

6. $\int [f(x)]^2 \cdot f'(x) dx = ?$

Yanıt / Answer : $\frac{1}{3}[f(x)]^3 + C$

7. $\int \frac{x dx}{(x^2 + 1)^2} = ?$

Yanıt / Answer : $\frac{-1}{2(x^2 + 1)} + C$

8. $\int \frac{\sec^2 x}{1 + \tan x} dx = ?$

Yanıt / Answer : $\ln(1 + \tan x) + C$

9. $\int \frac{2x^2 + 6x - 2}{x^2 + 2x - 2} dx = ?$

Yanıt / Answer : $2x + \ln|x^2 + 2x - 2| + C$

10. $\int \frac{x^2 + 2x - 1}{3 - x} dx = ?$

Yanıt / Answer : $-\frac{1}{2}x^2 - 5x - 14\ln|3 - x| + C$

11. $\int x \cdot e^{-x^2} dx = ?$

Yanıt / Answer : $-\frac{1}{2}e^{-x^2} + C$

12. $\int \frac{e^x dx}{1 + e^{2x}} = ?$

Yanıt / Answer : $\arctan(e^x) + C$

13. $\int \frac{x - 3}{x^2 - 4x - 5} dx = ?$

Yanıt / Answer : $\ln \sqrt{(x+1)^2 |x-5|} + C$

14. $\int \frac{2x^2 + 4x + 1}{2x^2 + 1} dx = ?$

Yanıt / Answer : $x + \ln|2x^2 + 1| + C$

15. $\int_9^{16} \frac{dx}{\sqrt{x+x}} = ?$

Yanıt / Answer : $\ln \frac{25}{16}$

16. $\int_1^2 5^{-2x+3} dx = ?$

Yanıt / Answer : $\frac{12}{5\ln 5}$

17. $\int \frac{2x+1}{2x-1} dx = f(x), \quad f(1)=1 \Rightarrow f(3) = ?$

Yanıt / Answer : $3 + \ln 5$

18. $\int_1^3 x \cdot y \cdot dy = 9 \Rightarrow x = ?$

Yanıt / Answer : $\frac{9}{4}$

19. $f'(x) = 4x^3 - 3x^2 - 1$ ve $f(1) = 2 \Rightarrow f(3) = ?$

Yanıt / Answer : 54

20. $\int (2x-1) \cdot f(x) dx = x^3 + x^2 - 3x + c \Rightarrow f(3) = ?$

Yanıt / Answer : 6

21. $\int_1^2 x^2 \cdot \ln x dx = ?$

Yanıt / Answer : $\frac{2}{3} \ln 2$

22. $\int_0^{e-1} \frac{x-1}{x+1} dx = ?$

Yanıt / Answer : $e - 3$

23. $\int_0^{\frac{\pi}{2}} x \cdot \sin x dx = ?$

Yanıt / Answer : 1

24. $\int (x + f(x)) dx = x^2 + ax + b, \quad f(3) = 5 \Rightarrow a = ?$

Yanıt / Answer : 2

25. $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx = ?$

Yanıt / Answer : $\frac{1}{4} + \frac{\pi}{8}$

26. $f(x) = \int_0^{\cos x} t \, dt \Rightarrow f\left(\frac{\pi}{6}\right) = ?$

Yanıt / Answer : $-\frac{\sqrt{3}}{4}$

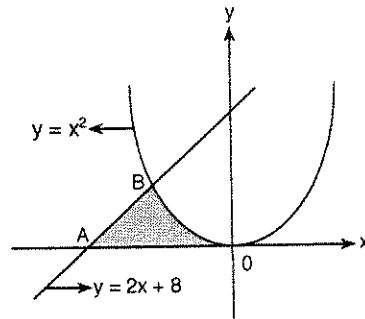
27. $\int \frac{1}{x \cdot \ln x} \, dx = ?$

Yanıt / Answer : $\ln(\ln x) + C$

28. $f^{-1}(x) = \frac{3x-1}{x+4} \Rightarrow \int_{-1}^1 d(f(x)) = ?$

Yanıt / Answer : $\frac{13}{4}$

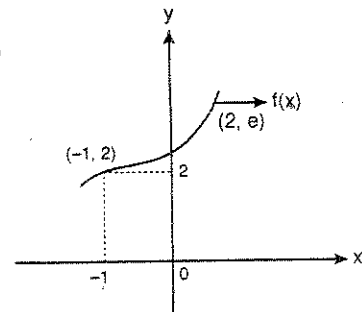
29.



Taralı alan / The shaded area = ? cm²

Yanıt / Answer : $\frac{20}{3}$

30. $\int_{-1}^2 \frac{f'(x)}{f(x)} \, dx = ?$



Yanıt / Answer : $\ln \frac{e}{2}$

1. $\int_1^2 \frac{2x^3+1}{x^2} dx = ?$

- A) 3 B) $\frac{7}{2}$ C) 4 D) $\frac{9}{2}$ E) 5

2. $f'(x) = 12x^3 - 6x + 1$, $f(-1) = 5$, $f(1) = ?$

- A) 5 B) 6 C) 7 D) 8 E) 9

3. $\int (3x-2) f(x) dx = 2x^3 + \frac{5}{2}x^2 - 6x + C$
 $\Rightarrow f(-1) = ?$

- A) -2 B) -1 C) 0 D) 1 E) 2

4. $\int_1^2 \frac{dx}{3x+1} = ?$

- A) $\frac{1}{2} \ln 14$ B) $\frac{1}{4} \ln \frac{7}{3}$ C) $\frac{1}{4} \ln \frac{3}{7}$
 D) $\frac{1}{3} \ln \frac{4}{7}$ E) $\frac{1}{3} \ln \frac{7}{4}$

5. $\int \left(\frac{x+1}{x} \right) dx = ?$

- A) $x + \ln |x| + C$ B) $x - \ln |x| + C$
 C) $x \cdot \ln |x| + C$ D) $\frac{x}{\ln |x|} + C$
 E) $\frac{\ln |x|}{x} + C$

6. $\int_0^1 \frac{3x dx}{x+1} = ?$

- A) $\ln \frac{e^3}{2}$ B) $\ln \frac{e^3}{8}$ C) $\ln \frac{e}{2}$
 D) $\ln \frac{e}{8}$ E) $\ln 8e$

7. $\int_2^4 xy dy = 12 \Rightarrow x = ?$

- A) 2 B) 3 C) 4 D) 5 E) 6

8. $\int_1^e \frac{\ln x}{x} dx = ?$

- A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

9. $\int_0^1 \frac{dx}{x^2+1} = ?$

- A) $\frac{\pi}{12}$ B) $-\frac{\pi}{8}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{5}$ E) $\frac{\pi}{4}$

10. $\int_0^1 \frac{3x^3}{x^2+3} dx = ?$

- A) $\frac{3}{2} \ln \frac{e}{4}$ B) $\frac{3}{2} \ln \frac{27e}{64}$ C) $\frac{3}{2} \ln \frac{e}{4}$
D) $\frac{3}{2} \ln \frac{3}{4}$ E) $\frac{3}{2} \ln \frac{1}{4}$

11. $\int_0^a (4x-5) dx = 12 \Rightarrow a = ?$

- A) 0 B) 1 C) 2 D) 3 E) 4

12. $\int_0^1 (x^2 + e^x) dx = ?$

- A) $e + \frac{4}{3}$ B) $e - \frac{4}{3}$ C) $e - \frac{2}{3}$
D) $e + \frac{2}{3}$ E) $e - \frac{3}{4}$

13. $\int_0^{\frac{\pi}{2}} \sin^2 x \cdot \cos x dx = ?$

- A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) 0 D) $-\frac{1}{3}$ E) $-\frac{1}{6}$

14. $b-a=5, \int_a^b (2x+1) dx = 25 \Rightarrow b = ?$

- A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) 4 E) $\frac{9}{2}$

15. $\int_0^2 (x-1)(x+2) dx = ?$

- A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) 1

16. $\int_0^1 \left(\frac{1-\sqrt{x}}{\sqrt{x}} \right) dx = ?$

- A) 1 B) 2 C) 3 D) 4 E) 5

17. $\int_0^1 x^2 (x^3+2)^2 dx = ?$

- A) 1 B) $\frac{4}{3}$ C) $\frac{19}{9}$ D) 2 E) $\frac{7}{3}$

18. $\int_0^{\frac{\pi}{2}} \cos^2 x dx = ?$

- A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$ E) $\frac{3\pi}{3}$

Yanıtlar / Answers

1. B	2. C	3. D	4. E	5. A	6. B
7. A	8. D	9. E	10. B	11. E	12. C
13. A	14. E	15. D	16. A	17. C	18. B

1. $f(x) = \frac{1}{x+2} \Rightarrow \int_2^3 d(f^{-1}(x)) = ?$

- A) $-\frac{1}{12}$ B) $-\frac{1}{6}$ C) $-\frac{1}{3}$
 D) $\frac{1}{3}$ E) $\frac{1}{6}$

2. $\int_{-1}^2 (2x+1)(x^2+x+1) dx = ?$

- A) 16 B) 20 C) 24 D) 28 E) 32

3. $\int_{-3}^3 (x + |x|) dx = ?$

- A) 1 B) 3 C) 6 D) 9 E) 12

4. $\int_{-2}^3 |x^2 - 2x| dx = ?$

- A) 8 B) $\frac{25}{3}$ C) $\frac{26}{3}$ D) 9 E) $\frac{28}{3}$

5. $\int_0^{\frac{\pi}{4}} (\cos x + \sin x) dx = ?$

- A) 1 B) $\frac{\sqrt{2}}{2}$ C) $\sqrt{2}$ D) $2\sqrt{2}$ E) 4

6. $\int_0^{\frac{\pi}{4}} \sin x \cdot \sin 2x dx = ?$

- A) $\sqrt{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{2}}{3}$
 D) $\frac{\sqrt{2}}{4}$ E) $\frac{\sqrt{2}}{6}$

7. $\int_0^{\frac{3}{2}} \frac{dx}{9+4x^2} = ?$

- A) $\frac{\pi}{6}$ B) $\frac{\pi}{9}$ C) $\frac{\pi}{12}$ D) $\frac{\pi}{24}$ E) $\frac{\pi}{30}$

8. $\int_0^{\frac{\pi}{2}} \cos^2 x dx = ?$

- A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$ E) 8

9. $\int_1^{e^2} x \cdot \ln x \cdot dx = ?$

- A) $\frac{e^2 - 1}{4}$ B) $\frac{4e^4 - e^4 + 1}{4}$ C) $\frac{4e^4 - e^2}{4}$
 D) $\frac{3e^2 + 1}{4}$ E) $\frac{e^4 - 4e^2 + 1}{4}$

10. $\int_0^1 \frac{2x}{3x+4} dx = ?$

- A) $\frac{3}{2} (1 + \ln 256)$ B) $\frac{1}{3} (1 + \ln 64)$
 C) $\frac{2}{3} (1 + \ln 8)$ D) $\frac{2}{3} (1 + \frac{4}{3} \cdot \ln \frac{4}{7})$
 E) $\frac{2}{3} (1 + \ln 2)$

11. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{1 - \cos 2x} dx = ?$

- A) $\frac{\sqrt{6}+2}{2}$ B) $\frac{\sqrt{6}-1}{4}$ C) $\frac{\sqrt{6}+1}{2}$
 D) $\frac{\sqrt{6}-1}{2}$ E) $\frac{\sqrt{6}-2}{2}$

12. $\int_0^{\sqrt{3}} \frac{x^2-1}{x^2+1} dx = ?$

- A) $\sqrt{3} - \frac{\pi}{3}$ B) $\sqrt{3} - \frac{\pi}{2}$ C) $\sqrt{3} - \frac{\pi}{6}$
 D) $\sqrt{3} - \frac{2\pi}{3}$ E) $\sqrt{3} - \frac{\pi}{4}$

13. $\int_0^{\frac{\pi}{2}} x \cdot \cos x dx = ?$

- A) $\pi - 1$ B) $\frac{\pi}{2} - 1$ C) $\frac{\pi}{2} - 2$
 D) $\pi - 2$ E) $\frac{\pi}{3} - 1$

14. $\int_1^{e^2} \frac{\ln x}{x} dx = ?$

- A) -4 B) -2 C) 2 D) 4 E) 6

15. $\int_0^{\ln 3} \frac{e^x}{e^x+1} dx = ?$

- A) $\ln 2$ B) $\ln 4$ C) $\ln \frac{1}{2}$
 D) $\ln 2 - \frac{1}{4}$ E) $\ln 8$

16. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = ?$

- A) $\frac{\pi}{24}$ B) $\frac{\pi}{12}$ C) $\frac{\pi}{6}$
 D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$

17. $\int_1^2 \frac{2}{x^2+2x} dx = ?$

- A) $\ln \frac{1}{4}$ B) $\ln \frac{1}{2}$ C) $\ln \frac{3}{2}$
 D) $\ln \frac{5}{2}$ E) $\ln 3$

Yanıtlar / Answers

1. B	2. C	3. D	4. E	5. A	6. E
7. D	8. C	9. B	10. D	11. E	12. D
13. C	14. B	15. A	16. B	17. C	

1. $\int (x^2 + 3x)^2 \cdot (2x+3) dx = ?$

A) $\frac{(x^2 + 3x)^3}{3} + x^2 + C$

B) $(x^2 + 3x)^3 + C$

C) $\frac{(x^2 + 3x)^3}{3} + C$

D) $\frac{(2x+3)^3}{3} + C$

E) $4(x^2 + 3x)^3 + C$

2. $\int \frac{x dx}{(x^2 + 4)^3} = ?$

A) $\frac{-1}{(x^2 + 4)^2} + C$

B) $\ln(x^2 + 4) + C$

C) $\arctan(2x) + C$

D) $\frac{-1}{4(x^2 + 4)^2} + C$

E) $(x^2 + 4)^{-3} + C$

3. $\int \frac{\sin x}{\cos^2 x} dx = ?$

A) $\sec x + C$

B) $\tan x + C$

C) $\cot x + C$

D) $\operatorname{cosec} x + C$

E) $\sin x + C$

4. $\int \frac{e^x - e^{-x}}{2} dx = ?$

A) $\frac{1}{2} (e^x - e^{-x}) + C$

B) $(e^x + e^{-x}) + C$

C) $\frac{1}{2} (e^x + e^{-x}) + C$

D) $(e^x - e^{-x}) + C$

E) $(e^{2x} + e^{-2x}) + C$

5. $\int \frac{x}{x+1} dx = ?$

A) $\ln|x+1| + C$

B) $x + \ln|x+1| + C$

C) $\frac{1}{2} \ln|x+1| + C$

D) $x \cdot \ln|x+1| + C$

E) $x - \ln|x+1| + C$

6. $\int \frac{dx}{x^2 + 6x + 10} = ?$

A) $\frac{1}{2} \arctan(x+3) + C$

B) $\arctan(x+3) + C$

C) $2 \arctan(x+3) + C$

D) $\arcsin(x+3) + C$

E) $\arccos(x+1) + C$

7. $\int \frac{dx}{\sqrt{4 - (x-3)^2}} = ?$

A) $\arcsin(x-3) + C$

B) $\frac{1}{2} \arcsin x + C$

C) $\frac{1}{2} \arcsin\left(\frac{x+3}{2}\right) + C$

D) $\frac{1}{2} \arcsin\left(\frac{x-3}{2}\right) + C$

E) $\arcsin\left(\frac{x-3}{2}\right) + C$

8. $\int \frac{dx}{9x^2 + 4} = ?$

A) $\arctan \frac{3x}{4} + C$

B) $\frac{1}{3} \arctan \frac{3x}{2} + C$

C) $\frac{1}{6} \arctan \frac{3x}{2} + C$

D) $\frac{1}{4} \arctan \frac{x}{2} + C$

E) $\frac{1}{9} \arctan \frac{2x}{3} + C$

9. $\int \frac{5dx}{x^2-3x-4} = ?$

- A) $5\ln|x^2-3x-4| + C$ B) $\ln\left|\frac{x-4}{x+1}\right| + C$
 C) $\frac{1}{5}\ln\left|\frac{x-4}{x+1}\right| + C$ D) $\ln|x^2-3x-4| + C$
 E) $\arctan\left|\frac{x-4}{x+1}\right| + C$

10. $\int_1^{\sqrt{3}} \frac{2x+1}{x^2+1} dx = ?$

- A) $\ln 4 + \frac{\pi}{6}$ B) $\ln 2 + \frac{\pi}{2}$ C) $\ln 4 + \frac{\pi}{12}$
 D) $\ln 2 + \frac{\pi}{12}$ E) $\ln 2 + \frac{\pi}{3}$

11. $\int_{-2}^0 x\sqrt{2x^2+1} dx = ?$

- A) -10 B) -5 C) $-\frac{13}{3}$ D) $-\frac{7}{2}$ E) $\frac{14}{3}$

12. $\int_0^{\frac{\pi}{4}} \sqrt{1-\cos 2x} dx = ?$

- A) $\sqrt{2} + 1$ B) $\frac{\sqrt{2}}{2} + 1$ C) $\sqrt{2} - 1$
 D) $\frac{\sqrt{2}}{2} - 1$ E) $2\sqrt{2}$

13. $\int_e^{e^2} \frac{dx}{x(\ln x)^2} = ?$

- A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$ E) $\frac{3}{4}$

14. $\int_0^4 |x-2| dx = ?$

- A) -2 B) -4 C) 1 D) 2 E) 4

15. $\int_1^2 \frac{\ln x}{x} dx = ?$

- A) $\ln 4$ B) $(\ln 2)$ C) $(\ln 2)^2$
 D) $\frac{1}{2} (\ln 2)^2$ E) $\frac{1}{4} (\ln 2)^2$

16. $\int_1^2 \frac{2x^3-3x^2+1}{x^2} dx = ?$

- A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

17. $\int_{1-e}^2 \frac{dx}{x+e} = ?$

- A) e B) $1 + \ln 2$ C) $\ln(2+e)$
 D) $2 + \ln 2$ E) 2e

Yanıtlar / Answers

1. C	2. D	3. A	4. C	5. E	6. B
7. E	8. C	9. B	10. D	11. C	12. C
13. C	14. E	15. D	16. A	17. C	

1. $\int 3x^2 + 2\sqrt{x} + 4 \, dx = ?$

- A) $x^3 + 4\sqrt{x^3} + 4x + C$ B) $3x^3 + \sqrt{x} + 4x + C$
 C) $x^3 + \frac{4}{3}\sqrt{x^3} + 4x + C$ D) $x^3 + 3\sqrt{x^3} + 4x + C$
 E) $x^3 + 3\sqrt{x^3} + 4 + C$

2. $\int \frac{3x^2 + 4}{(x^3 + 4x)^2} \, dx = ?$

- A) $\ln|x^3 + 4x| + C$ B) $\ln|3x^2 + 4| + C$
 C) $x^3 + 4x + C$ D) $-x^3 - 4x + C$
 E) $-\frac{1}{x^3 + 4x} + C$

3. $\int x^2(x^3 + 1)^2 \, dx = ?$

- A) $\frac{(x^3 + 1)^3}{3} + C$ B) $\frac{(x^3 + 1)^3}{9} + C$
 C) $\frac{(x^3 + 1)^2}{3} + C$ D) $\frac{(x^3 + 1)^2}{9} + C$
 E) $3x + 1 + C$

4. $\int (e^{3x} + 5^{2x}) \, dx = ?$

- A) $e^{3x} + 2 \cdot 5^{2x} \ln 5 + C$ B) $\frac{e^{3x}}{3} + \frac{2 \cdot 5^{2x}}{\ln 5} + C$
 C) $\frac{e^{2x}}{3} + \frac{5^{2x}}{2 \ln 5} + C$ D) $\frac{e^{3x}}{3} + \frac{5^{2x}}{2 \ln 5} + C$
 E) $e^{3x} + \frac{\ln 5 \cdot 5^{2x}}{2} + C$

5. $\int \frac{\cos x}{2 + \sin x} \, dx = ?$

- A) $\frac{1}{(2 + \sin x)^2} + C$ B) $\ln(2 + \sin x) + C$
 C) $2 \ln(2 + \sin x) + C$ D) $\frac{1}{\ln(2 + \sin 2x)} + C$
 E) $\ln(\cos x)^2 + C$

6. $\int \frac{\cos x}{1 + \sin^2 x} \, dx = ?$

- A) $\text{Arctan} x + C$ B) $\text{Arccot} x + C$
 C) $\text{Arctan}(\sin x) + C$ D) $\text{Arctan}(\cos x) + C$
 E) $\text{Arcsin} x + C$

7. $\int \frac{\ln(\sin x) \cdot \cos x}{\sin x} \, dx = ?$

- A) $\frac{\ln|\cos x|}{2} + C$ B) $\frac{\ln|\sin x|}{2} + C$
 C) $\frac{\ln^2|\sin x|}{2} + C$ D) $\frac{\ln|\arcsin x|}{2} + C$
 E) $\frac{\ln|\sin x| + \cos x}{2} + C$

8. $\int \frac{2x+1}{x} \, dx = ?$

- A) $2x + \ln|x| + C$ B) $2 + \ln|x| + C$ C) $\frac{2}{x^2} + C$
 D) $\frac{x^2}{2} + \ln|x| + C$ E) $2x + \ln x^2 + C$

9. $\int (x^2y + e^xy + e^y) dy = ?$

- A) $\frac{x^{3y}}{3} + e^xy + xe^y + C$ B) $\frac{x^2y^2}{2} + e^xy + e^y + C$
 C) $\frac{y^2(x^2 + e^x)}{2} + e^y + C$ D) $x^2y + \frac{e^xy^2}{2} + C$
 E) $\frac{x^2 + y^2}{2} + \frac{e^xy^2}{2} + xe^y + C$

10. $\int \cos(\cos^2x) \cdot \sin 2x dx = ?$

- A) $-\sin(\cos^2x) + C$ B) $-\cos(\cos^2x) + C$
 C) $\sin(\cos^2x) + C$ D) $-2\sin(\cos x) + C$
 E) $-\sin(\cos x) + C$

11. $\int (2x+1)f(x) dx = 2x^3 + 5x^2 + 10x + C \Rightarrow f(-1) = ?$

- A) -12 B) -8 C) -6 D) 3 E) 12

12. $\int \frac{x^2 \arctan x^3}{1+x^6} dx = ?$

- A) $\frac{(\arctan x^3)}{3} + C$ B) $\frac{\arctan x^2}{6} + C$
 C) $\frac{(\arctan x^3)^2}{3} + C$ D) $\frac{(\arctan x^3)^2}{6} + C$
 E) $\frac{(\arctan x^3)^2}{2} + C$

13. $\int \frac{1}{\sqrt{81-9x^2}} dx = ?$

- A) $\text{Arctan} \frac{x}{3} + C$ B) $\frac{\text{Arctan} \frac{x}{3}}{9} + C$
 C) $\frac{\text{Arctan} \frac{x}{3}}{2} + C$ D) $\frac{\text{Arctan} \frac{x}{3}}{3} + C$
 E) $\frac{1}{3} \text{Arc sin} \frac{x}{3} + C$

14. $\int \frac{1}{x^2+6x+10} dx = ?$

- A) $\text{Arcsin}(x+3) + C$ B) $2\text{Arctan}(x+3) + C$
 C) $\frac{\text{Arctan}(x+3)}{2} + C$ D) $\frac{\text{Arcsin}(x+3)}{2} + C$
 E) $\text{Arctan}(x+3) + C$

15. $\int \frac{e^{\tan x}}{\cos^2 x} dx = ?$

- A) $\arctan(\cos x) + C$ B) $\tan(\cos x) + C$
 C) $e^{\sin^2 x} + C$ D) $e^{\cos^2 x} + C$
 E) $e^{\tan x} + C$

16. $\int \frac{2^x}{2^{x+1} + 4^x + 1} dx = ?$

- A) $\frac{\text{Arctan} 2^x}{\ln 2} + C$ B) $\frac{\sin 2^x}{\ln 2} + C$
 C) $\frac{\text{Arctan}(2^x + 1)}{\ln 2} + C$ D) $\frac{1}{\ln 2 \cdot (2^x + 1)} + C$
 E) $-\frac{1}{\ln 2 \cdot 2^x} + C$

Yanıtlar / Answers

1. C	2. E	3. B	4. D	5. B	6. C	7. C	8. A
9. C	10. A	11. C	12. D	13. E	14. E	15. E	16. D

1. $\int \frac{1}{4+64x^2} dx = ?$

- A) $\frac{\text{Arc sin } 16x}{16} + C$ B) $\frac{\text{Arc tan } 4x}{16} + C$
 C) $\frac{\text{Arc tan } 64x}{16} + C$ D) $\frac{\text{Arc tan } 16x}{64} + C$
 E) $\frac{\text{Arc sin } 4x}{4} + C$

2. $\int \frac{-\sin x}{\sqrt{2-\cos^2 x}} dx = ?$

- A) $\sqrt{2} \text{Arc tan} \left(\frac{\cos x}{\sqrt{2}} \right) + C$
 B) $\sqrt{2} \text{Arc sin} \left(\frac{\cos x}{\sqrt{2}} \right) + C$
 C) $\text{Arc sin} \left(\frac{\cos x}{\sqrt{2}} \right) + C$
 D) $\frac{\text{Arc sin} \left(\frac{\cos x}{\sqrt{2}} \right)}{\sqrt{2}} + C$
 E) $-\sqrt{2} \text{Arc sin} \left(\frac{\cos x}{\sqrt{2}} \right) + C$

3. $\int x \cdot \log_5 e^{x^2} dx = ?$

- A) $\frac{x^4}{4 \ln 5} + C$ B) $\frac{x^3}{3} \log_5 e + C$
 C) $\frac{x^4}{4} \cdot \log_5 e^{x^2} + C$ D) $2 \log_5 e^{x^2} + C$
 E) $\frac{2 \cdot \log_5 e^{x^2}}{\ln 5} + C$

4. $0 < x < \frac{\pi}{2}$

$\int 3 \sin^2(\pi - x) \cdot \cos(\pi + x) dx = ?$

- A) $\frac{\sin^3 x}{3} + C$ B) $\frac{\cos^3 x}{3} + C$ C) $\frac{\tan^2 x}{2} + C$
 D) $-\frac{\sin^3 x}{3} + C$ E) $-\sin^3 x + C$

5. $0 < x < \frac{\pi}{2}$

$\int \sqrt{36-4x^2} dx = ?$

- A) $x \cdot \sqrt{9-x^2} + \arcsin \frac{x}{3} + C$
 B) $9x \cdot \sqrt{9-x^2} + \arcsin \frac{x}{3} + C$
 C) $x \cdot \sqrt{9-x^2} + 9 \arcsin \frac{x}{3} + C$
 D) $x \cdot \sqrt{9-x^2} + \frac{x}{3} + C$
 E) $x \cdot \sqrt{9-x^2} + \arcsin x + C$

6. $\int \frac{x^2+3x+1}{x^2+x} dx = ?$

- A) $\arctan |x| + C$ B) $\ln |x^2 + x| + C$
 C) $x + \ln |x^2 + x| + C$ D) $\frac{x^2 + \ln |x^2 + x|}{x} + C$
 E) $(2x+1) \cdot \ln |x| + C$

7. $\int \frac{1-\sqrt{x}}{2\sqrt{x}} dx = ?$

- A) $\frac{(1-\sqrt{x})^2}{2} + C$ B) $\sqrt{x} - \frac{1}{2}x + C$
 C) $(\sqrt{x}+1)^2 + C$ D) $\frac{x+\sqrt{x}}{2} + C$
 E) $\frac{1+\sqrt{2}x}{2} + C$

8. $\int \frac{dx}{\sqrt{x+x}} = ?$

- A) $2\sqrt{x} - \ln(1+\sqrt{x}) + C$ B) $2 \ln(1+\sqrt{x}) + C$
 C) $x - \ln(1+\sqrt{x}) + C$ D) $\frac{\ln(1+\sqrt{x})}{2} + C$
 E) $2 \ln \left(\frac{1+(x)}{x} \right) + C$

9. $f(x) = \int (\cos^2 x + \cos 2x - \frac{1}{2}) dx$ ve (and) $f(0) = 0$

$\Rightarrow f(\frac{\pi}{4}) = ?$

- A) 1 B) $\frac{3}{4}$ C) $\frac{7}{4}$ D) $\frac{9}{2}$ E) $\frac{9}{4}$

10. $\int \frac{1}{x^2 + 5x + 6} dx = ?$

- A) $\ln\left(\frac{x+1}{x+2}\right) + C$ B) $\ln\left(\frac{x+3}{x+2}\right) + x + C$
 C) $\ln\left(\frac{x+2}{x+3}\right) + C$ D) $\ln\left(\frac{x^3}{3} + \frac{5x^2}{2} + 6x\right) + C$
 E) $\ln(x^2 + 5x + 6) + C$

11. $\int \frac{1}{x^2 + 8x + 17} dx = ?$

- A) $\text{Arctan}(x+4) + C$ B) $\text{Arccot}(x+4) + C$
 C) $\ln(x+4) + C$ D) $\text{Arcsin}(x+4) + C$
 E) $\ln(x+4)^2 + C$

12. $\int \frac{\cot x}{\ln|\sin x|} dx = ?$

- A) $\ln|\sin x| + C$ B) $\ln|\ln|\sin x|| + C$
 C) $\ln|\cos x| + C$ D) $\frac{1}{\ln|\sin x|} + C$
 E) $1 + \ln|\sin x| + C$

13. $\int \sin^6 x \cdot \cos^3 x dx = ?$

- A) $\frac{\sin^7 x}{7} + \frac{\cos^4 x}{4} + C$ B) $\sin^9 - \sin^7 + C$
 C) $\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$ D) $\frac{\sin^7}{7} - \frac{\cos^8 x}{9} + C$
 E) $\frac{\tan^7}{7} + C$

14. $\int \frac{\sqrt[3]{2x+7} - \sqrt{2x+7}}{2x+7} dx = ?$

- A) $\sqrt[6]{2x+7} + C$
 B) $\sqrt[6]{(2x+7)^2} - \sqrt[6]{(2x+7)^3} + C$
 C) $\frac{3}{2} \sqrt[3]{(2x+7)} - \sqrt{(2x+7)} + C$
 D) $\frac{3}{2} \sqrt{(2x+7)} - \sqrt[3]{(2x+7)} + C$
 E) $\frac{3}{2} \sqrt[3]{(2x+7)^2} + \sqrt{(2x+7)} + C$

15. $\int \frac{1}{\sqrt{x-x^2}} dx = ?$

- A) $\arcsin \frac{x}{2} + C$ B) $2 \arcsin \frac{3x}{2} + C$
 C) $\arctan \sqrt{x} + C$ D) $\arcsin \sqrt{x} + C$
 E) $2 \arcsin \sqrt{x} + C$

Yanıtlar / Answers

1. B	2. C	3. A	4. E	5. C	6. C	7. B	8. B
9. B	10. C	11. A	12. A	13. C	14. C	15. E	

1. $F(x) = \int_0^{\sqrt{x}} (t+2)^{\frac{1}{2}} dt \Rightarrow F'(4) = ?$

- A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

2. $F(x) = \int_1^{2x} \cos(t^2) dt \Rightarrow F'(\frac{\sqrt{2\pi}}{4}) = ?$

- A) $-\frac{1}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) 0 D) $\frac{\sqrt{2}}{2}$ E) $\frac{1}{2}$

3. $F(x) = \int_{\sin x}^0 \frac{dt}{2+t} \Rightarrow F'(\frac{5\pi}{6}) = ?$

- A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) $\frac{\sqrt{3}}{3}$ E) $\frac{\sqrt{3}}{5}$

4. $F(x) = \int_x^{x^2} \ln(t) dt \Rightarrow F'(e) = ?$

- A) $4e-1$ B) $2e-1$ C) 3
D) 2 E) $4e+1$

5. $F(x) = \int_x^{\ln x} e^t dt \Rightarrow F'(1) = ?$

- A) $1+e$ B) $1-e$ C) e
D) 0 E) 1

6. $F(x) = \int_x^{x^2} \frac{\ln t}{2+\ln^2 t} dt \Rightarrow F'(e) = ?$

- A) $\frac{2e}{3}$ B) $\frac{2}{3}$ C) $\frac{2e+1}{4}$
D) $\frac{2e-1}{3}$ E) $\frac{2e+1}{4}$

7. $\int \sin x \cdot f(x) dx = \sin^2 x - \cos^2 x + x \Rightarrow f(\frac{\pi}{3}) = ?$

- A) 2 B) $\frac{2\sqrt{3}}{3}$ C) $2 + \frac{2\sqrt{3}}{3}$
D) $\frac{2\sqrt{3}+1}{3}$ E) $\sqrt{3} + \frac{2}{3}$

8. $\int x \cdot f(x) dx = x^3 + 3ax^2 - 2x + 4$

$f(2) = 9 \Rightarrow a = ?$

- A) 2 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{2}{3}$ E) $\frac{3}{4}$

9. $\int f(x) dx = \arctan(2x) + \sin \frac{\pi}{2} x - \frac{\sqrt{2}}{4} \pi x$
 $\Rightarrow f(\frac{1}{2}) = ?$

- A) 1 B) $1 + \frac{\pi\sqrt{2}}{2}$ C) $1 - \frac{2\sqrt{2}}{4}$
D) $\frac{1}{2}$ E) $1 + \frac{\pi}{2}$

10. $F(x) = \int_0^{2x} \sin(\frac{\pi}{4} t) dt \Rightarrow F'(1) = ?$

- A) 1 B) $\frac{3}{2}$ C) 2 D) 3 E) $\frac{7}{2}$

11. $\int \sec^2 x f(x) dx = \tan^2 x (1 + \tan x) + x \Rightarrow f\left(\frac{\pi}{4}\right) = ?$

- A) 3 B) 5 C) $\frac{11}{2}$ D) $\frac{13}{2}$ E) $\frac{15}{2}$

12. $\frac{df(x)}{dx} = 3x - 4$ ve (and) $f(-1) = \frac{13}{2} \Rightarrow f(x) = ?$

- A) $y = \frac{3x^2}{2} - 4x + 2$ B) $y = \frac{3x^2}{2} - 4x + 1$
 C) $y = \frac{3x^2}{2} - 4x + 3$ D) $y = \frac{1}{2} - x^3 - 2x^2 - 2$
 E) $y = x^2 - 4x + 6$

13. $\frac{d^2 f(x)}{dx^2} = -x^2 - 2x$, $\frac{df(1)}{dx} = 0$, $f(1) = 1 \Rightarrow f(x) = ?$

- A) $f(x) = -\frac{1}{6}x^3 - \frac{1}{3}x^2 + \frac{4}{3}x + 2$
 B) $f(x) = -\frac{1}{12}x^3 - \frac{1}{6}x^2 + \frac{4}{3}x + 1$
 C) $f(x) = -\frac{1}{12}x^3 - \frac{1}{3}x^2 + \frac{4}{3}x - \frac{1}{24}$
 D) $f(x) = -\frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{4}{3}x + \frac{1}{12}$
 E) $f(x) = -\frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{4}{3}x + \frac{1}{12}$

14. $\frac{d^3 f(x)}{dx^3} = e^x + 1$, $\frac{d^2 f(0)}{dx^2} = 1$, $\frac{df(0)}{dx} = 2$, $f(0) = 3$

$\Rightarrow f(x) = ?$

- A) $f(x) = e^x + \frac{x^2}{3} + x + 9$
 B) $f(x) = e^x + \frac{x^2}{3} + 3x + 6$
 C) $f(x) = 2e^x + \frac{x \cdot e^x}{3} + x^2 \cdot e^x + 12$
 D) $f(x) = e^x + \frac{e^{2x}}{3} + x \cdot e^x + 6$
 E) $f(x) = e^x + \frac{1}{6}x^3 + x + 2$

15. $\frac{df(x)}{dx} = x^2 - x$, $f(3) = 4 \Rightarrow f(1) = ?$

- A) 2 B) $\frac{3}{2}$ C) $\frac{1}{3}$ D) $-\frac{2}{3}$ E) -2

16. $F(x) = \int (x^2 - 1)e^{x^3 - 3x} dx$, $F(\sqrt{3}) = 3 \Rightarrow F(x) = ?$

- A) $e^{x^3 - 3x} + \frac{3}{4}$ B) $\frac{1}{3}e^{x^3 - 3x} + \frac{8}{3}$
 C) $(x^2 - 1)e^{x^2 - 1} + 7$ D) $\frac{1}{2}e^{x^2 - 1} + \frac{9}{4}$
 E) $\frac{1}{3}e^{x^3 - 3x} + \frac{11}{3}$

17. $f(x) = \frac{2x}{x^2 + 5}$, $f(1) = \frac{1}{3}$ ve (and) $\int d\left(\frac{2x}{x^2 + 5}\right) = \frac{3}{7}$

$\sum x = ?$

- A) 4 B) $\frac{13}{3}$ C) $\frac{14}{3}$ D) 5 E) $\frac{7}{3}$

18. $f(x) = \int \frac{1}{x \ln x} dx$ ve (and) $f(e) = 6 \Rightarrow f(x) = ?$

- A) $\ln x + 6$ B) $\ln^2 x + 6$ C) $2 \ln x + 3$
 D) $\ln(\ln x) + 6$ E) $x^2 \ln x + 9$

Yanıtlar / Answers

1. C	2. C	3. E	4. A	5. B	6. D
7. C	8. D	9. A	10. C	11. C	12. B
13. D	14. E	15. D	16. B	17. C	18. D

1. $\int_{-2}^1 |x| dx = ?$

- A) $\frac{7}{2}$ B) 4 C) 3 D) $\frac{11}{4}$ E) $\frac{5}{2}$

2. $\int_1^3 (x+1)e^{x^2+2x} dx = ?$

- A) $\frac{e^3}{2}(e^{12}-1)$ B) $\frac{e^2}{3}(e^9-1)$ C) $\frac{e^{12}}{3}-1$
D) $e^{15}-15$ E) $\frac{1}{2}e^3(e^9+1)$

3. $\int_4^5 \frac{2}{(x-3)^3} dx = ?$

- A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{2}{3}$ E) $\frac{3}{4}$

4. $\int_{\frac{1}{2}}^3 \frac{1}{x^2} dx = ?$

- A) $-\frac{1}{3}$ B) $-\frac{1}{6}$ C) 2 D) $\frac{5}{3}$ E) $\frac{5}{4}$

5. $\int_3^4 \frac{e^{\ln x}}{x} dx = ?$

- A) $2 \cdot \ln 2$ B) 0 C) 3 D) 2 E) 1

6. $\int_0^2 x^2 e^{x^3} dx = ?$

- A) $\frac{1}{3}(e^8-1)$ B) $\frac{e}{3}(e^6-1)$ C) $\frac{1}{3}e^9-1$
D) e^6-1 E) $\frac{1}{6}(e^6-1)$

7. $\int_1^2 x \ln x dx = ?$

- A) $\ln 2 - 1$ B) $\frac{5}{4}$ C) $\ln 4 - 3$
D) $\ln 4 - \frac{3}{4}$ E) $\ln 8 - \frac{1}{2}$

8. $\int_0^1 \frac{dx}{4-x^2} = ?$

- A) $\frac{1}{2} \ln 3$ B) $\frac{3}{4}$ C) $\frac{\pi}{3}$ D) $\frac{1}{4} \ln\left(\frac{3}{4}\right)$ E) $\frac{\pi}{6}$

9. $\int_0^2 \frac{dx}{4+x^2} = ?$

- A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{8}$

10. $\int \frac{dx}{\sqrt{4-(x-1)^2}} = ?$

- A) $\arcsin\left(\frac{x-1}{2}\right) + C$ B) $\arccos\left(\frac{x-1}{2}\right) + C$
C) $\operatorname{arcsec}\left(\frac{x-1}{2}\right) + C$ D) $\arctan\left(\frac{x-1}{4}\right) + C$
E) $\operatorname{arccosec}\left(\frac{x-1}{4}\right) + C$

11. $\int \frac{dx}{x \cdot \sqrt{a^2 + x^2}} = ?$

A) $\frac{x}{a + \sqrt{a^2 - x^2}} + C$

B) $\frac{1}{a} \ln \left| \frac{x}{a + \sqrt{x^2 + a^2}} \right| + C$

C) $\frac{1}{a^2} \arcsin \left(\frac{x}{a + \sqrt{x^2 + a^2}} \right) + C$

D) $\frac{ax}{a + \sqrt{x^2 + a^2}} + C$

E) $\frac{1}{a^2} \ln \left| \frac{2x}{a + \sqrt{x^2 + a^2}} \right| + C$

12. $\int_{2\sqrt{2}}^3 \frac{dx}{x \sqrt{9 - x^2}} = ?$

A) $\ln(\sqrt{6} + \sqrt{2}) - \ln 2$

B) $\frac{1}{2} \ln(2 + \sqrt{3})$

C) $\ln \sqrt{6} - \ln \sqrt{3}$

D) $\ln \sqrt{3} - 1$

E) $\ln \sqrt{2} + 1$

13. $\int_0^{\frac{\pi}{6}} \sec 2t \, dt = ?$

A) $\ln(\sqrt{6} + \sqrt{2}) - \ln 2$

B) $\ln(\sqrt{3} + 1) - \ln 2$

C) $\ln \sqrt{6} - \ln \sqrt{3}$

D) $\ln \sqrt{3} - 1$

E) $\ln \sqrt{2} + 1$

14. $\int \tan^2 4x \, dx = ?$

A) $\frac{1}{4} \tan^2 4x + x + C$

B) $\frac{1}{4} \tan^2 4x + 4x - x + C$

C) $\frac{1}{16} \tan 4x - x + C$

D) $\frac{1}{16} \tan^2 4x - 2x + C$

E) $\frac{1}{4} \tan 4x - x + C$

15. $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \frac{dx}{x \ln(3x)} = ?$

A) 3

B) $\frac{1}{2} \ln 3$

C) $\ln 6$

D) 6

E) $\frac{3}{4}$

16. $\int \tan^3 x \cdot \sec x \, dx = ?$

A) $\frac{1}{3} \tan^3 x - \tan x + C$

B) $\frac{1}{4} \tan^3 x + \tan x + C$

C) $\frac{1}{4} \sec^4 x + \tan x + C$

D) $\frac{1}{3} \sec^3 x - \sec x + C$

E) $\frac{1}{2} \sec^2 x + \sec x + C$

Yanıtlar / Answers

1. E	2. A	3. E	4. D	5. E	6. A
7. D	8. D	9. E	10. A	11. B	12. B
13. B	14. E	15. C	16. D		

1. $\frac{dy}{dx} = \frac{1}{9+x^2} \Rightarrow \int_0^3 dy = ?$

- A) $\frac{\pi}{6}$ B) $\frac{\pi}{9}$ C) $\frac{\pi}{12}$ D) $\frac{\pi}{18}$ E) $\frac{\pi}{24}$

2. $\int \frac{\sin x}{2 - \cos x} dx = ?$

- A) $2 + \cos x + C$
 B) $\ln(2 - \cos x) + C$
 C) $\ln(2 - \sin x) + C$
 D) $2 - \ln(\cos x) + C$
 E) $\ln(4 - \tan x) + C$

3. $\int \frac{x \, dx}{1-x^2} = ?$

- A) $-\frac{1}{2} \ln|1-x^2| + C$
 B) $\frac{1}{2} \ln|1-x^2| + C$
 C) $\frac{1}{4} \ln|1-x| - \frac{1}{4} \ln|1+x| + C$
 D) $\ln x^2 - 1 + C$
 E) $2 \ln x - 1 + C$

4. $\int (\ln x)^2 \frac{dx}{x} = ?$

- A) $\frac{1}{2} \ln^2 x + c$ B) $\frac{1}{3} (\ln x)^3 + c$ C) $\frac{1}{3} \ln x^3 + c$
 D) $\frac{1}{6} (\ln x)^2 + c$ E) $3 \ln x + c$

5. $\int e^{\sin x} \cos x \, dx = ?$

- A) $e^{\cos x} + C$ B) $e^{\tan x} + C$ C) $e^{\sin x} + x + C$
 D) $e^{\cos x + 1} + C$ E) $e^{\sin x} + C$

6. $\int_e^{e^2} \frac{dx}{x \ln x} = ?$

- A) $\frac{1}{2}$ B) 2 C) $-\ln 2$ D) $\ln 2$ E) $2 \ln 2$

7. $\int_0^{\ln 2} e^{-2x} dx = ?$

- A) $\frac{3}{4}$ B) $\frac{3}{8}$ C) $\frac{3}{16}$ D) $\frac{5}{6}$ E) $\frac{5}{8}$

8. $\int_0^{\frac{\pi}{6}} (\cos x) \cdot 4^{-\sin x} dx = ?$

- A) $\frac{1}{\ln 16}$ B) $\frac{1}{\ln 8}$ C) $\frac{1}{\ln 4}$ D) $\frac{1}{\ln 2}$ E) $2 \ln 2$

9. $\int_0^3 x(e^{x^2-1}) dx = ?$

- A) $\frac{e^9 + 1}{2e}$ B) $\frac{e^9 - 1}{e}$ C) $\frac{e^9 - 1}{2e}$
 D) $\frac{e^6 - 1}{e}$ E) $\frac{1 - e^9}{4}$

10. $\int_0^1 (e^x + 1) dx = ?$

- A) 1 B) e C) $e + 1$ D) $e - 1$ E) $2e$

11. $\int \sec x \, dx = ?$

- A) $\ln|\sec x + \tan x| + C$ B) $\ln|\sec x - \tan x| + C$
 C) $\sec x \cdot \tan x + C$ D) $\sec x + \tan x + C$
 E) $\frac{1}{2} |\sec x + \tan x| + C$

12. $\int_0^{\frac{\pi}{6}} \sec x \, dx = ?$

- A) $\frac{2}{3}$ B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $-\frac{1}{3}$ E) $-\frac{2}{3}$

13. $\int_{\frac{\pi}{6}}^{\pi} \tan^3 2x \, dx = ?$

- A) $\frac{3}{2}$ B) $\frac{\ln 2 + 3}{2}$ C) $\frac{\ln 4 - 3}{4}$
 D) $\ln 4 + 1$ E) $\ln 2 - 1$

14. $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{2 + \tan x} \, dx = ?$

- A) 0 B) 1 C) 2
 D) $\ln 2 - \ln 3$ E) $\ln 3 - \ln 2$

15. $\int_{\frac{\pi}{18}}^{\frac{\pi}{6}} \sin^2 3x \cdot \cos 3x \, dx = ?$

- A) $\frac{3}{8}$ B) $\frac{5}{16}$ C) $\frac{9}{32}$ D) $\frac{11}{72}$ E) $\frac{7}{72}$

16. $\int_0^{\frac{\pi}{3}} \frac{\sin^3 x}{\cos^2 x} \, dx = ?$

- A) $\frac{3}{2}$ B) 1 C) $\frac{3}{4}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

17. $\int_0^1 \frac{dx}{\sqrt{4-x^2}} = ?$

- A) $\frac{1}{2}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{12}$

18. $\int_0^2 \frac{x \, dx}{4+x^2} = ?$

- A) $\ln \sqrt{2}$ B) $\ln 2$ C) $\ln^3 2$
 D) $2 \ln 2$ E) $3 \ln 2$

19. $\int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\sqrt{1-\cos^2 \theta}} \, d\theta = ?$

- A) $\frac{3}{2}$ B) $\frac{3}{4}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{12}$ E) $\frac{\pi}{18}$

Yanıtlar / Answers							
1. C	2. B	3. A	4. B	5. A	6. D	7. B	8. A
9. C	10. B	11. A	12. C	13. C	14. E	15. E	16. D
17. D	18. A	19. C					

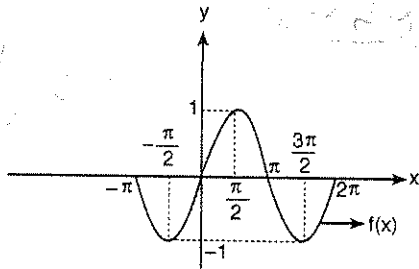
1. $\int_0^1 x^2 e^x dx = ?$

- A) $5e + 1$ B) $2e - 1$ C) $e - 2$
D) $e + 2$ E) $e - 4$

2. $\int_0^{2\sqrt{3}} \frac{(x^2 + 4) \cdot x}{(x^2 + 4)^2} dx = ?$

- A) 2 B) $\frac{5}{2}$ C) $4\ln 2$ D) $\ln 2$ E) $\ln \sqrt{2}$

3.



$f(x) = \sin x \Rightarrow \int_{-\pi}^{2\pi} \sin x dx = ?$

- A) 6 B) 4 C) 3 D) 2 E) 1

4. $\int_1^2 (2x + 5) dx = ?$

- A) 6 B) 8 C) 9 D) 10 E) 12

5. $\int_0^1 (x^2 - 2x + 2) dx = ?$

- A) $\frac{7}{3}$ B) $\frac{5}{2}$ C) 3 D) $\frac{9}{2}$ E) 5

6. $\int_{-1}^1 (x+1)^2 dx = ?$

- A) 3 B) $\frac{7}{2}$ C) 4 D) $\frac{17}{4}$ E) $\frac{13}{3}$

7. $\int_0^2 \sqrt{4x+1} dx = ?$

- A) 3 B) $\frac{7}{2}$ C) 4 D) $\frac{17}{4}$ E) $\frac{13}{2}$

8. $\int_0^1 \frac{dx}{(2x+1)^3} = ?$

- A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) $\frac{4}{9}$ D) $\frac{1}{12}$ E) $\frac{1}{15}$

9. $\int_a^b (4x+1) dx = 30$

$b - a = 6 \Rightarrow a + b = ?$

- A) 12 B) 8 C) 5 D) 4 E) 2

10. $\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{\cos^2 2x} dx = ?$

- A) 3 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) $\frac{1}{4}$

11. $\int_0^{\pi} \sin^2 x \, dx = ?$

- A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

12. $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx = ?$

- A) $\frac{\pi}{8}$ B) $\frac{\pi+2}{8}$ C) $\frac{\pi-2}{8}$
D) $\frac{\pi+4}{16}$ E) $\frac{\pi-4}{16}$

13. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \, dx = ?$

- A) $\sqrt{2}$ B) $1 - \sqrt{2}$ C) $\sqrt{2} - 1$
D) $\sqrt{2} + 1$ E) $2\sqrt{2} - 1$

14. $\int_0^1 \arcsin x \, dx = ?$

- A) $\frac{\pi+1}{2}$ B) $\frac{\pi-2}{2}$ C) $\frac{\pi-2}{2}$
D) $\frac{\pi-1}{3}$ E) $\frac{\pi-4}{4}$

15. $\int_0^1 (x^2 + 3)x \, dx = ?$

- A) $\frac{7}{4}$ B) $\frac{11}{2}$ C) $\frac{13}{2}$ D) $\frac{17}{4}$ E) 6

16. $\int_0^{\frac{\pi}{2}} \sqrt{1 + \cos x} \, dx = ?$

- A) 1 B) $\sqrt{2} + 1$ C) $\sqrt{3} - 1$
D) 2 E) $2\sqrt{2}$

17. $\int_0^{\frac{\pi}{3}} \frac{dx}{\cos^4 x} = ?$

- A) $\sqrt{3} + 1$ B) $2\sqrt{3}$ C) $\sqrt{3} - 1$
D) $2\sqrt{3} + 1$ E) $2 + \sqrt{3}$

18. $\int_0^{\frac{1}{\sqrt{6}}} x \cdot \cos(\pi x^2) \, dx = ?$

- A) $\frac{1}{2\pi}$ B) $\frac{1}{4}$ C) $\frac{1}{4\pi}$ D) $\frac{1}{6}$ E) $\frac{1}{8}$

19. $\int_0^1 \frac{3x+1}{\sqrt[3]{3x^2+2x+3}} \, dx = ?$

- A) $3 - \frac{3\sqrt[3]{9}}{4}$ B) $\frac{11}{2}$ C) 5 D) $\frac{7}{4}$ E) 1

Yanıtlar / Answers

1. C	2. D	3. E	4. B	5. A	6. B	7. E	8. C
9. E	10. C	11. A	12. B	13. C	14. B	15. A	16. D
17. B	18. C	19. A					